

## PROPOSED RENORMALIZATION GROUP ANALYSIS OF NONLINEAR BRAIN DYNAMICS AT CRITICALITY

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Perception is characterized by the formation of spatiotemporal patterns of neural activity that embody mental categories of the material events provided by the senses. The patterns are constructed by modifications of the background activity, which is maintained and self-regulated at criticality, such that all frequencies and wavelengths coexist in neural activity, from the atomic level to the whole brain. Pattern formation depends on energy dissipation and occurs by phase transition upon the coincidence of two events, both endogenous. One event is the null spike in the Rayleigh noise, which is generated by mutual excitation and then band pass filtered by feedback inhibition. The frequency-specific drop in background amplitude enhances the signal-to-noise ratio of sensory-driven activity in each sensory sample taken by an action-perception cycle under limbic control. The other event is the sensory-selected activity from a Hebbian nerve cell assembly constituting reactivation of a memory of experience from past learning that is mobilized by the limbic system. The neural mechanisms of the phase transition that mediates perception may be subject to description in terms of a renormalization group based on systematic segmentation of the temporal spectra of various measures of brain activity.

### 1. Introduction

The action-perception cycle of Merleau-Ponty occurs across scales. An act begins at the macroscopic brain level with the intent to collect information from the world by thrusting the body into the environment while predicting the microscopic sensory consequences of that action and assimilating to them by learning [2]. Brains make hypotheses (predictions) by extrapolating from remembered experiences, which are stored in the modified strengths of synaptic

connections that mediate the neural interactions. Strengthening of connections in associative learning creates Hebbian nerve cell assemblies that amplify foreground inputs. Habituation to unreinforced input sharpens the assemblies and de-amplifies background inputs. Each assembly sustains an attractor in cortical dynamics that implements a learned category of sensation. The collection of learned inputs that might be expected consequent to an act of observation are predicted by a landscape of attractors. The synaptic changes in the interdigitated cell assemblies support spatiotemporal pattern formation. The patterns are seen in segments (frames) of EEG that sustain phase-locked oscillatory potentials having carrier frequencies in the beta (12-30 Hz), gamma (30-80 Hz) and epsilon (80-250 Hz) ranges. The patterns are spatial amplitude modulations (AM) of the common waveform [3,4]. The processes of forming and accessing the memory stores are mediated by pre-stimulus background activity [5]. Hence background activity offers a key to unlock the enigma of perception. We believe renormalization group theory [9,10] can help do this.

## **2. Nonlinear coupling among neurons forming interactive populations**

Cortical neurons have high packing density ( $10^4/\text{mm}^3$ ) and high spatial divergence/convergence ( $10^4$  synapses/neuron), yet sparse connectivity (1% of neurons within the radius of dendritic branches of most neurons (0.1 mm). Cortical neurons are >80% excitatory and <20% inhibitory; >90% of cortical synapses are by axons originating intracortically; <10% originate subcortically from sensory neurons. Therefore, cortical neural interactions are dominated by excitation among excitatory neurons. Each neuron sustains regenerative activity through positive feedback by multisynaptic serial transmission of spikes, so it interacts with its surround, not with any countable neurons in networks. The feedback to a neuron from a single spike by that neuron can be modeled as a modified 1-D diffusion process [1]. Spike formation is limited by refractory periods; each spike triggers a recovery process during which the neuron cannot fire or can only do so with diminished probability. Yet connection densities are so great that neurons maintain self-sustained activity. The spike trains from mutual excitation yield interval histograms that start at zero probability, rise to a peak, and decay exponentially to a non-zero steady-state baseline, so they can be described as modified Poisson processes manifesting random walks. Summation over a local interactive excitatory population (designated K1e [1]) gives a pulse density function of axonal output and a wave density function of dendritic output, each with a Gaussian amplitude density distribution and a  $1/f^\alpha$  power spectral density (PSD) in log-log coordinates with exponent  $\alpha$  ranging between 2 and 3 (brown vs. black noise [4] with slopes -2 and -3) depending on the sensitivities of synapses at dendrites (relating mainly to  $\text{Ca}^{++}$ ) and refractory periods (relating mainly to  $\text{K}^+$ ) at the trigger zones of axons [1].

Owing to these properties the background activity of a K1e population can be described as that of a lattice with such fine grain that it can be treated as a continuum, and its dynamics can be modeled with derivatives given by ordinary

differential equations (ODE) [1]. The densities are locally limited by the refractory periods, not by thresholds or inhibitory interneurons, and they are globally adapted by neuromodulatory chemicals supplied by subcortical nuclei to appropriate levels of behavioral arousal. The continuum is bounded between scales of neurons (1 ms for spike duration, 1  $\mu\text{m}$  for axon diameter) and scales of brains (cortical circumference  $\sim 0.5$  m, lifespan  $\sim 70$  years,  $10^{13}$  neurons and  $10^{17}$  synapses in each hemisphere). The coupling across scales between ions and spikes is calculated with the Hodgkin-Huxley equations. That between spikes and waves is calculated from the normalized probability of spike firing conditional on the amplitude, time delay and relative frequency of wave amplitude [1]. Sections through this 3-D table of normalized conditional pulse probability density in time and amplitude yield pulse probability waves in designated frequency ranges in time and the sigmoid curve [2] showing the nonlinear population dependence of pulse density on wave density. The lower asymptote is imposed by threshold. The upper asymptote  $Q_m$  is imposed by refractory periods. The asymmetry reflects the exponential increase in firing probability of single neurons as the wave amplitude approaches threshold from below. Other couplings across EEG frequencies are explored with the high temporal resolution enabled by the Hilbert transform [3].

### 3. Proposed application of renormalization group theory

The property of the cortical dynamics that suggests the suitability of renormalization group analysis is the unity of events across scales. Yet wave packets emergent by phase transition [3,5,7], are nearly stationary for  $\sim 60$ -160 ms; the transitions by which they form are shorter by two orders of magnitude; the processes of metabolic energy replenishment are longer by two orders of magnitude, because concentration gradients of ions across membranes drive the dendritic and axonal. Hence the coupling of neural events to metabolism and heat dissipation, while essential, is remote, as evidenced by the disparity in scales between EEG/MEG measurements in ms and fMRI measurements in s.

It is the coupling of activities at frequencies across spectra that is in need of exploration. Given the typical brown noise " $1/f^2$ " spectrum [1,4] in awake resting subjects, a convenient way to segment the spectra for cross-spectral study is to divide them in multiples of 2 [10]. Supposing that the sample interval is 2 ms for a digitizing rate of 500/s, the Nyquist frequency is 250 Hz. This segmentation of the spectrum conforms fairly closely to the clinical partitioning in pass bands but given in multiples of the Nyquist frequency.

clinical name	empirical (Hz)	theoretical (Hz)	multiples of Nyquist
epsilon	80-250	100-200	1/2.5 – 1/1.25
high gamma	50-80	50-100	1/5.0 - 1/2.5
low gamma	30-50	25-50	1/10 - 1/5.0
beta	12-30	12.5-25	1/20 - 1/10
alpha	7-12	6.25-12.5	1/40 - 1/20
theta	3-7	3.125-6.25	1/80 - 1/40

The opportunity for renormalization group theory is analysis of coupling between bands. The coupling between action potentials and epsilon activity is documented in the chatter of repetitive spike trains in mutual excitation [1]. The coupling between pulse densities and beta/gamma oscillations is documented by calculating pulse probability conditional on wave amplitude [1,2]. Gamma-high-theta and beta-low-theta coupling has been shown in field potentials from cats and rabbits [4,5]; beta-theta/alpha coupling has been shown in human scalp EEG. These empirical relationships call for theory. Other properties in need of explanation are the repeated resynchronization over relatively immense correlation distances; de-amplification of noise-driven activity outside the range of self-regulation; amplification of input in spontaneous null spikes with approach to a limit cycle attractor; and the neural mechanisms by which Hebbian and non-Hebbian modification of synaptic connectivity supports the construction of attractor landscapes. Of special significance here is the Kondo problem, in which very sparse iron impurities at the atomic level in copper modify the macroscopic properties of the metal [9]. While not of great importance in itself, the problem has spawned numerous studies owing to its tractability. The analogy to the effects on cortical dynamics of modified synapses seen as ‘impurities’ enhances the likelihood that renormalization group theory will be useful, if it can be used to model the multiple ‘impurities’ that support the attractor landscapes from which diverse AM patterns emerge in the retrieval of memories from experience.

## References

1. W. J. Freeman, *Mass Action in the Nervous System*, New York, Academic (1975/2004).
2. W. J. Freeman, *How Brains Make Up Their Minds*, New York, Columbia UP (2001).
3. W. J. Freeman, Origin, structure, and role of background EEG activity. Part 3. Neural frame classification. *Clin. Neurophysiol.* 116 (5), 1118-1129 (2005).
4. W. J. Freeman, Origin, structure, and role of background EEG activity. Part 4. Neural frame simulation. *Clin. Neurophysiol.* 117(3), 572-589 (2006).
5. W. J. Freeman, Proposed cortical ‘shutter’ in cinematographic perception. Invited Chapter in: *Neurodynamics of Cognition and Consciousness*. R. Kozma and L. Perlovsky (eds.): New York: Springer, August (2007, in press).
6. W. J. Freeman and G. Vitiello, Nonlinear brain dynamics as macroscopic manifestation of underlying many-body field dynamics. *Physics Life Rev.* 3, 93-118 (2006). <http://dx.doi.org/10.1016/j.plrev.2006.02.001>
7. R. Kozma, M. Puljic, P. Balister, B. Bollabás and W. J. Freeman, Phase transitions in the neuropercolation model of neural populations with mixed local and non-local interactions. *Biol. Cybern.* **92**: 367-379 (2005).
7. M. Schroeder, *Fractals, Chaos, Power Laws*. W. H. Freeman (1991).
8. K. G. Wilson, The renormalization group: Critical phenomena and the Kondo problem. *Rev. Mod. Phys.* 47: 773-840 (1975).
9. K. G. Wilson, The renormalization group and critical phenomena. *Rev. Mod. Phys.* 55: 583-600 (1983).