CHAOTIC RESONANCE — METHODS AND APPLICATIONS FOR ROBUST CLASSIFICATION OF NOISY AND VARIABLE PATTERNS

ROBERT KOZMA and WALTER J. FREEMAN*

Division of Neurobiology, University of California at Berkeley, LSA 142, Berkeley, CA 94720-3200, USA

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A fundamental tenet of the theory of deterministic chaos holds that infinitesimal variation in the initial conditions of a network that is operating in the basin of a low-dimensional chaotic attractor causes the various trajectories to diverge from each other quickly. This “sensitivity to initial conditions” might seem to hold promise for signal detection, owing to an implied capacity for distinguishing small differences in patterns. However, this sensitivity is incompatible with pattern classification, because it amplifies irrelevant differences in incomplete patterns belonging to the same class, and it renders the network easily corrupted by noise. Here a theory of stochastic chaos is developed, in which aperiodic outputs with $1/f^2$ spectra are formed by the interaction of globally connected nodes that are individually governed by point attractors under perturbation by continuous white noise. The interaction leads to a high-dimensional global chaotic attractor that governs the entire array of nodes. An example is our spatially distributed KIII network that is derived from studies of the olfactory system, and that is stabilized by additive noise modeled on biological noise sources. Systematic parameterization of the interaction strengths corresponding to synaptic gains among nodes representing excitatory and inhibitory neuron populations enables the formation of a robust high-dimensional global chaotic attractor. Reinforcement learning from examples of patterns to be classified using habituation and association creates lower dimensional local basins, which form a global attractor landscape with one basin for each class. Thereafter, presentation of incomplete examples of a test pattern leads to confinement of the KIII network in the basin corresponding to that pattern, which constitutes many-to-one generalization. The capture after learning is expressed by a stereotypical spatial pattern of amplitude modulation of a chaotic carrier wave. Sensitivity to initial conditions is no longer an issue. Scaling of the additive noise as a parameter optimizes the classification of data sets in a manner that is comparable to stochastic resonance. The local basins constitute dynamical memories that solve difficult problems in classifying data sets that are not linearly separable. New local basins can be added quickly from very few examples without loss of existing basins. The attractor landscape enables the KIII set to provide an interface between noisy, unconstrained environments and conventional pattern classifiers. Examples given here of its robust performance include fault detection in small machine parts and the classification of spatiotemporal EEG patterns from rabbits trained to discriminate visual stimuli.

*Author for correspondence.

E-mail: wfreeman@socrates.berkeley.edu
1. Introduction

When recurrent neural networks are used for pattern classification, they are commonly parameterized to have a landscape of point attractors, each with its basin of attraction to provide for generalization over incomplete patterns and degradation by noise [Hopfield, 1982; Amari, 1977; Kohonen, 1972; Anderson, 1977; Grossberg, 1980]. Less commonly limit cycle attractors have been used [Eisenberg et al., 1988; Ishii et al., 1996]. Deterministic chaotic attractors have appeared ill-suited for pattern classification owing to their sensitivity to initial conditions, though Tsuda [1992] has suggested this as an application for pattern classification, using "itinerant trajectories" over successions of "attractor ruins", each attractor being destroyed as soon as it is reached, so that the network continuously remains unstable, as in a search mode.

Several groups [Kaneko, 1986, 1990; Aihara et al., 1990; Perrone & Basi, 1995; Wang, 1996; Schuster & Stemmler, 1997; Borisuyk & Borisuyk, 1997; Minai & Anand, 1998] have proposed the use of networks of nodes for pattern classification, each node being governed by a deterministic chaotic attractor such as those of Lorenz, Chua or Rössler (a twist-flip map) [Brown & Chua, 1991]. Most modelers have relied on nearest-neighbor coupling in one-dimensional [Andreyev et al., 1996] or two-dimensional arrays [Chua et al., 1995]. Emphasis has been given to the "control" of chaos (i.e. its suppression) [Carroll & Pecora, 1993; Ditto et al., 1990], or to synchronization of multiple nodes by periodic inputs [Nakagawa, 1998] or input-dependent strengthening of connections gains [Yao & Freeman, 1991; Schiff et al., 1994].

The work reported here is based on a biological finding in the olfactory system, which is that the chaotic attractor is a mesoscopic property of the entire system. The neurons and local populations comprising the system are governed by a broad array of zero and nonzero point attractors, to which the components return, if the connections between them are cut surgically or pharmacologically [Freeman, 2000]. The importance of this finding for pattern classification is that each pattern to be classified is expressed by a two-dimensional array of point inputs from olfactory receptors in the nose, and the class of the pattern is expressed by the relation of each point to every other point, which constitutes a global property. Therefore, classification must be based on a global landscape of multiple basins, each with a global chaotic attractor corresponding to a class of patterns, not on selection from a cluster of local attractors.

This insight led to the development of a hierarchical collection of K-sets, named in honor of Aharon Katchalsky, a pioneer in studies of collective behavior of neurons [Freeman, 1975]. The KO set is the basic building block, describing a neuron population with a linear second order ordinary differential equation (ODE) in series with a gain coefficient at the input to represent synapses, and a sigmoid static nonlinear function at the output to represent the collective trigger zones. KI sets represent either all excitatory or all inhibitory populations in positive feedback. KII sets have both kinds of populations to give negative as well as the two kinds of positive feedback. The KIII set has multiple layers comprising KO, KI and KII sets coupled with both positive and negative feedback through distributed delay lines. The KIII network reported here has been designed and parameterized to simulate the observed dynamic behavior of the olfactory system, including a distributed layer of 64 globally interconnected elements to represent columns formed by glomeruli in the olfactory bulb.

The KIII model has been tested for verisimilitude in two respects: first, by simulation of the wave forms and statistical properties of the EEG pulse densities, and evoked potentials (impulse responses) recorded from the several parts of the olfactory system, which has been accomplished as already reported [Yao et al., 1991; Shimoide & Freeman, 1995; Aradi et al., 1995], and second, by simulation of its capability for pattern classification. Substantial difficulties have been encountered with the second test, when the constraints of the first test are maintained, as expressed in a set of ten criteria for the properties of the state variables [Freeman et al., 1997].

First among these problems was the lack of shadowing trajectories [Hamel et al., 1987; Fryzka & Zohdy, 1992]. Because the olfactory system and its KIII model both have Lyapunov exponents for which the real parts fluctuate above and below zero [Freeman, 1975, 1992], the KIII model cannot solve the shadowing problem, owing to the resulting numerical instabilities [Dawson et al., 1994]. This difficulty was resolved by use of alternative numerical techniques for solving the ordinary difference equations used to approximate ODEs in digital simulations of the KIII model, that were
mathematically equivalent but gave different chaotic solutions, owing to the sensitivity to initial conditions [Chang et al., 1998].

The typical number of nodes each having a second-order difference equation exceeded 330, giving rise to a plethora of attractors and the problem of attractor crowding [Wiesenfeld & Hadley, 1989]. The large number led to decrease in the size of basins until they approached the size of the double-precision 64-bit rational numbers used in numerical simulations (giving 1 part in $10^{15.4}$ precision), so that in simulated run times on the order of a second (at 1 ms digitizing steps) the KIII network exited its assigned basin of a chaotic attractor and entered the basin of a neighboring point or quasi-periodic attractor, from which in digital embodiments there was no escape other than terminating the runs. A related problem was the exquisite sensitivity of the system to changes in selected parameters on the order of $\pm 10^{-12}$, which exposed the extreme delicacy of the deterministic system.

These difficulties were resolved by introducing additive noise that was modeled on known sources of noise in the olfactory system [Freeman et al., 1997; Chang & Freeman, 1998]. This step introduced an alternative to deterministic chaos, which is referred to here as “stochastic chaos” [Freeman, 2000], in which the extreme sensitivity of the KIII network to changes in state variables and parameters is greatly ameliorated, without loss and even with enhancement of the simulations of the observed activity patterns in the olfactory system [Freeman et al., 1997]. While there is at present no mathematical proof by which to support this practice, intuitively it appears that the noise serves to broaden or coalesce the otherwise pointillist basins in the landscape, or to fuzzify the trajectory in a manner perhaps related to the “pseudo-periodic drifting” described by Pecora and Carroll [1991].

In the present report this addition is exploited to demonstrate substantial improvement in the capacity of the KIII network for pattern classification. In particular, a crippling weakness of the KIII network was encountered after it had been given a chaotic basal state and then been trained to classify input patterns. There was a marked tendency for the network on input to leave the basin of the global attractor for the background state, as expected, but, instead of entering the basin of another global attractor for a desired class, to disintegrate into a collection of local quasi-periodic attractors, in effect designating the example instead of the class. In order to remedy this defect, systematic exploration was done of the parameters of the feedback gains in the KII sets comprising the three main oscillatory networks in the olfactory system [Kozma & Freeman, 1999a]. The domains of acceptable parameter values necessary to provide the zero, positive (excitatory), and negative (inhibitory) values of the point attractors governing the olfactory bulb, nucleus and cortex were identified, and the noise levels were optimized to improve substantially the classificatory power of the system.

In Sec. 2 the topological structure of the KIII network is reviewed, and the representations of input and output are described. In Sec. 2.3 the state space of the KII components is graphed in order to show the domains specified by the gain parameter values that give the zero and nonzero point attractors derived from biological studies [Freeman, 1975, 2000a]. In Sec. 3 the learning rules used in the model for habituation and association are summarized. Section 4 reviews the criteria for the state variables in the KIII network to optimize the noise levels for the basal control state using an artificial data set. The efficacy of the dynamical chaotic memory system is demonstrated with two examples, one doing quality control on small machine parts, and the other classifying spatiotemporal EEG patterns that were derived by recording in the visual system of rabbits trained to discriminate visual stimuli, thus closing the loop by applying the KIII network to the biological data on which its design for classification was based.

**Glossary**

**Latin letters**

| A, B | Pattern classes to be analyzed |
| AM | Amplitude modulated pattern |
| AON | Anterior olfactory nucleus |
| CR | Chaotic resonance |
| G | Granulate cell layer |
| H0 | Null hypothesis of Student’s $t$-test |
| H1 | Alternative hypothesis of Student’s $t$-test |
| $h_{hab}$ | Habituation learning coefficient |
| $h_{hab-A}$ | Habituation learning coefficient of pattern A |
| $h_{hab-c}$ | Learning coefficient of continuous habituation |
| $h_{Heb}$ | Hebbian learning coefficient |
\( h_{Heb-B} \) Hebbian learning coefficient of pattern B

\( K \) Bias coefficient of Hebbian learning, \( 0 \leq K \leq 1 \)

KO, KI, KII K-set of zero-, first-, and second-order

KIII K-set of third-order; this is the highest order K-set in this study

M Mitral cell layer

OB Olfactory bulb

\( p \) Significance level of the statistical hypothesis test

PC, PPC Prepyriform cortex layer

\( P_1, P_2, \ldots, P_8 \) Gamma-filtered activity in the \( i \)th layer of the KIII model, \( i = 1, \ldots, 8 \)

\( P_M(i) \) Gamma-filtered mitral signal at the \( i \)th node

\( \bar{P}_M \) Spatial ensemble average mitral activity

PG Periglomerular cell layer

R Receptor cell layer

RMS Root mean square fluctuation intensity

SR Stochastic resonance

\( W_{M(i)\rightarrow M(j)} \) Gain parameter from node \( M(i) \) to \( M(j) \) modifiable in learning

\( W_{EE}, W_{II} \) Gain parameters between Excitatory–Excitatory, Inhibitory–Inhibitory,

\( W_{IE}, W_{EI} \) Excitatory–Inhibitory, and Inhibitory–Excitatory nodes, respectively

**Greek letters**

\( \Phi_M(i) \) Four-vector of the time-lagged gamma filtered mitral signal at the \( i \)th node

\( \sigma_c \) Standard deviation of the central noise at AON

\( \sigma_{RI} \) Standard deviation of the input noise

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2. Summary Description of KIII Architecture, Input, Output and KII Components

2.1. *KIII nodes, layers and connectivity modeled on the olfactory system*

The KIII network has a multilayer architecture corresponding to the two-dimensional arrays of sensory receptors R, periglomerular cells PG, olfactory bulb OB with mitral M and granule G layers, anterior olfactory nucleus AON and prepyriform cortex PC with excitatory and inhibitory layers and deep pyramidal nodes in the PC. This architecture is shown in the center insert of Fig. 1. Note that the present implementation will allow layers at the level of olfactory bulb and prepyriform cortex, as indicated in Fig. 1; the computer simulations in this work, however, are simplified by use of the lumped PC instead of the distributed PC. Also, the deep pyramidal layer is represented by a single lumped node. Odorants constituting sensory signals enter the system via R, where they are transduced to action potentials. Neural activity propagates through M and G layers, then to the AON and PC in parallel and on to the deep pyramidal cells. Lateral connections with negligible delays are dominant within the layers; global feedback connections between the OB, AON and PC layers are modeled with distributed delay lines. The OB generates a spatially coherent aperiodic carrier wave that is amplitude modulated to give spatial AM patterns. The AON appears to act as a chaotic controller. The PC provides the output of the olfactory system to other parts of the brain. Further details have been given [Chang & Freeman, 1998].

In the present software embodiment the R, PG and OB are represented by distributed KO, KI and KII sets each with 64 channels, whereas the AON and PC are represented by lumped KII sets. There are lateral connections within the M layer and within the G layer, but not within the R or PG layers. The negative feedback connections between M and G nodes are local in conformance to the extreme localization provided by the dendrodendritic synapses in the OB [Freeman, 1975]. The forward connections from the M layer to the AON and PC are convergent and with negligible delays. The feedback connections from the AON to the OB and from the PC to the AON and PC are divergent and have
Fig. 1. Illustration of the structure and dynamics of the KII sets embedded in the topology of the multilayer KIII network is shown to have layers of the periglomerular, mitral, granule, prepyriform and deep layers. In simulations reported here, a lumped KII set was used to simulate the prepyriform cortex and deep pyramidal cell layers. On the right-hand side, phase diagrams are shown for OB, AON and PC units (see Glossary). On the left-hand side, phase diagrams are shown as contour plots, with...
distributed delays that are modeled by second-order low-pass filters. The connections between R axons and the PG and M layers are modifiable by local homeostatic feedback [Chang & Freeman, 1998] for dynamic range control of the KII set in the bulb but have no role in learning. The excitatory connections within the M layer between M nodes and the excitatory connections from M nodes to G nodes are modifiable with learning, as described later.

2.2. *Input as a 1 × 64 column vector and output as a 4 × 64 vector*

The input is a continuous time series expressed in a 1 × 64 column vector, for which the 64 amplitudes are fixed in 200 ms time segments, and for which the max–min range is compressed between 0 and 1 (all positive values correspond to excitatory input). Compression in the bulb is by logarithmic conversion [Freeman, 1975] modeled by homeostatic feedback [Chang & Freeman, 1999], but that complicated simulation is not needed here. The background input is zero, and spatially patterned input is given as step functions lasting 200 ms at intervals of 200 ms. Continuous rectified Gaussian noise with zero mean and unit standard deviation is added to the input by random numbers, independently generated on each channel; the SD is scaled to between 1% and 5% of the range. Continuous Gaussian noise with zero mean and unit standard deviation and an excitatory bias, $K$, is given as input to the AON to simulate the input from many other parts of the forebrain to the olfactory system [Freeman et al., 1992].

The output of the KIII set is taken as a 1 × 64 column vector at the mitral level (not at the PC as in the brain) to express the AM patterns of the spatially coherent aperiodic carrier wave, specifically the input-induced oscillations in the gamma band (20 Hz to 80 Hz) across the 64 channels during the input-maintained state transition. The activity of the $i$th node in the mitral layer $P_M(i)$ is calculated as the root mean square (RMS) value of the gamma-filtered mitral signal over the period $[T_1 + 20, T_2]$, where $T_1$ and $T_2$ are the start and end time points of the presentation of the input pattern, after an initial transient period of 20 ms.

$$P_M(i) = \text{RMS}_{[T_1 + 20, T_2]} P_{320Hz,80Hz}(i),$$

$$i = 1, \ldots, 64$$

An alternative measure of mitral activity provides detailed information on the trajectory of the activity of the mitral nodes not captured by the average RMS value during the input-maintained phase transition. Trajectory information is expressed by a series of RMS values calculated for 40 ms intervals during the transient in the vector function $\Phi_M(i)$:

$$\Phi_M(i) = \begin{cases} \text{RMS}_{[T_1 + 40, T_1 + 80]} P_{320Hz,80Hz}(i) \\ \text{RMS}_{[T_1 + 80, T_1 + 120]} P_{320Hz,80Hz}(i) \\ \text{RMS}_{[T_1 + 120, T_1 + 160]} P_{320Hz,80Hz}(i) \\ \text{RMS}_{[T_1 + 160, T_1 + 200]} P_{320Hz,80Hz}(i) \end{cases}$$

$$i = 1, \ldots, 64$$

$\Phi_M(i)$ is a 4 × 64 vector containing the RMS values of the 64 nodes as segments along the trajectories. By increasing the dimension of vector $\Phi_M(i)$, additional details of the trajectories can be captured. It is disadvantageous to follow the trajectories in too much detail owing to computational overload. The use of four-dimensional $\Phi_M$ vectors can significantly improve classification performance, reflecting evolutionary change of the spatial AM patterns during the input-maintained phase transitions.

2.3. *Parameterization of the three KII sets in the KIII network*

The global chaotic landscape of the KIII set arises primarily through the interaction of the three KII sets, which are coupled by both positive and negative feedback through distributed delay lines, giving rise to the positive, zero and negative Lyapunov exponents in the linearized equations [Freeman, 1975, 2000a]. The characteristic frequencies of the three parts of the olfactory system are determined by measurement of time-ensemble averaged impulse responses of the OB, AON and PC in the small-signal, near-linear range of responses to perturbation. The impulse inputs are too brief to induce state transitions, and the responses can be closely approximated by damped cosines. The three complex frequencies are unequal and incommensurate. They are simulated by adjustment of the four gain parameters, $W_{EE}$, $W_{IE}$, $W_{EI}$ and $W_{II}$, that are required to describe the synaptic actions of excitatory and inhibitory neurons on the same and opposing kinds [Freeman, 1987], where E stands for excitatory node and I for inhibitory node.
Fig. 2. (a–c) Attractor regions in the KII set. The 3-D plots correspond to three gain values: \(W_{II}, W_{EE}\), and \(W_{EI}\) vary from 0 to 2, while \(W_{IE} = 0.4\) (fixed); (a) no bias: OB; (b) excitatory bias: AON; (c) inhibitory bias: PC. The total range of variation of gain parameters shown in this figure is from 0 to 2.
Detailed analysis of the dynamics of the olfactory system under diverse experimental conditions has revealed that the three parts, when disengaged, are governed by point attractors with differing values. That of the OB has zero offset, that of the AON has excitatory (positive) offset, and that of the PC has negative (inhibitory) offset [Freeman, 1987, 1992]. The impulse responses calculated by isolated KII sets are shown in the right panels of Fig. 1 as decaying oscillations. There are, however, important differences in the asymptotic convergence to fixed-point. The KII set can have a long-term bias manifesting a nonzero point attractor. Systematic variations of the gain parameters reveal regions in the state space in which the attractors are robust. Initially, the less sensitive inhibitory–excitatory and excitatory–inhibitory gains are fixed at \( W_{Ei} = 0.4 \) and \( W_{IE} = 0.4 \), respectively. By varying the remaining two gains, \( W_{EE} \) and \( W_{II} \), we build phase diagrams and identify parameter regions where zero offset, excitatory offset, and inhibitory offsets are observed for the KII set. Convergence is checked by showing that the numerical derivative goes to zero. The zero fixed point is \( 0 \pm 0.01 \), where the total range of the asymmetric sigmoid function is \(-1\) to \(+5\). Trajectories to nonzero points that go beyond the \( \pm 1 \) threshold are excluded.

The results are shown as contour plots on the left side of Fig. 1. Red color indicates the dominance of a given type of fixed point behavior, while blue shows the absence of the desired asymptotic behavior. Some parameter values correspond to crisp attractor regions; there are extended regions where the zero offset (OB-type) and inhibitory offset (PC-type) are observed. For the excitatory offset (AON-type), however, only a narrow basin region is found and there are significant overlaps among the attractors, depending on the initial conditions. This behavior resembles the partially ordered phase-type attractors (glassy or intermittent) in globally coupled lattices [Kaneko, 1990].

Figure 2 illustrates the above observations in a three-dimensional plot, where parameters \( W_{EE} \), \( W_{II} \), and \( W_{Ei} \) vary, while the least sensitive parameter, \( W_{IE} \), is fixed at 0.4. Red to brown colors emphasize different regions of the state space of the KII set that are occupied by the OB, AON and PC. There is an extended region where the unbiased attractor is dominant [Fig. 2(a)]. Also, the attractor region of the inhibitory bias state shown in Fig. 3(c) is quite large. The excitatory bias state, however, is limited to a narrow tube only, as it is seen after enhancement in Fig. 2(b). Once the gain parameters are fixed to place each KII set in its proper domain, the chaotic KIII dynamics remains stable under wide variations of input and robust under reinforcement learning [Kozma & Freeman, 1999a].

3. Learning Rules

The operation of the KIII dynamic memory can be described as follows. In the absence of stimuli the system is in a high-dimensional state of spatially coherent basal activity. The basal state is described by an aperiodic (chaotic) global attractor. In response to an external stimulus, the system is kicked out of the basal state into a local memory basin of an attractor wing. This wing is inferred to be of lower dimension than the basal attractor [Freeman, 1988] giving spatially patterned amplitude modulation (AM) of the coherent carrier wave. The system resides in this localized wing approximately for the duration of the stimulus and it returns to the basal state after the stimulus ends. This temporal burst process is given a duration of about 200 ms. The system memory is defined as the collection of basins and attractor wings of the KIII set, and a recall is the induction by a state transition of a spatiotemporal gamma oscillation with a spatial AM pattern. The AM patterns are determined by the connectivity within the M and G layers, which is shaped by reinforcement learning. Three processes are used to this end.

- Hebbian reinforcement learning of stimulus patterns; fast, one-step, irreversible.
- Habituation of background activity; slow, cumulative, reversible.
- Normalization of nodal activities to maintain overall stability; state variable and parameter optimization outside real time.

These learning processes exist in a subtle balance and their relative importance changes at various stages of the memory process. In the framework of this study, stable operation of the KIII memory network is achieved without using normalization. Therefore, this method now is used for initialization of the state variables to direct the KIII system to converge to the desired attractors with minimal transient time and reduced computational load. Further details of this method are given in the Appendix.
3.1. Habituation

Habituation is an automatic process in every primary sensory area that serves to screen out stimuli that are irrelevant, confusing, ambiguous or otherwise unwanted. It constitutes an adaptive filter to reduce the impact of environmental noise that is continuous and uninformative. It is continually updated in a form of learning, and it can be abruptly canceled ("dis-habituation") by novel stimuli and almost as quickly reinstituted ("re-habituation") if the novel stimuli are not reinforced. It is a central process that does not occur at the level of sensory receptors. It is modeled here by incremental weight decay that decreases the sensitivity of the KIII system to stimuli from the environment that are not designated as desired or significant by accompanying reinforcement. In accord with physiological processes [Freeman, 1975] it is automatic unless countermanded by reinforcement. The affected synapses are located not at the R synapses on M nodes but at the synapses of M nodes on other nodes in the OB. On each trial run the forward gains of the output connections from M nodes to other M nodes and G nodes are reduced, but only for those M nodes that have been excited by undesired inputs. Habituation is executed according to the rule:

\[
\begin{align*}
\text{IF } P_M(i) > \overline{P}_M \text{ THEN } & \quad W'_{M(i) \rightarrow M(j)} = h_{\text{hab}} \ast W_{M(i) \rightarrow M(j)} \\
& \quad W'_{M(i) \rightarrow G(j)} = h_{\text{hab}} \ast W_{M(i) \rightarrow G(j)} \\
& \quad i = 1, \ldots, S \\
\text{ELSE Do_Not_Change.}
\end{align*}
\]

Here \( P_M(i) \) denotes the activity of the \( i \)th node of the mitral layer, \( S \) is the number of M nodes receiving input on that trial, \( S < N; \bar{P}_G \) is the spatial ensemble average activity across the mitral layer; \( N = 64 \) is the layer size. The activity of a given node is calculated as the band-passed RMS value in the range 20 Hz to 80 Hz during the presentation of a given input pattern, as given by Eq. (1). \( W' \) stands for the weight after learning, while \( W \) is the original weight; \( h_{\text{hab}} \) is the learning constant of the habituation. Further details of the choice of the habituation learning parameter are given below.

3.2. Hebbian learning rule

The modified Hebbian reinforcement learning rule used here holds that pairs of M nodes that are coactivated by a stimulus accompanied by reinforcement have their connections strengthened. The rule differs from that for habituation in that the change in gain occurs only when a stimulus arriving at both M nodes is accompanied by a signal representing reinforcement. It is symmetric between the pair in both directions and it is irreversible. In the present implementation, the weights that connect two M nodes are increased from a fixed low value to a fixed high value, if the two nodes show high activation simultaneously, and the signal is given to make the change on that trial. In mathematical terms:

\[
\begin{align*}
\text{IF } P_M(i) > \overline{P}_M \text{ AND } P_M(j) > \overline{P}_M \text{ THEN } & \quad W'_{M(i) \rightarrow M(j)} = h_{\text{Heb}} \\
& \quad W'_{M(j) \rightarrow M(i)} = h_{\text{Heb}} \\
\text{ELSE Do_Not_Change.}
\end{align*}
\]

Notations of mitral activity \( P_M(i) \), the spatial average mitral activity \( \overline{P}_M \), and the connection weights are the same as in the case of habituation, as given above. By considering normally distributed weight space, it is clear that about half of the nodes satisfy each part of the Hebbian condition during a learning epoch. Therefore, approximately 25% of the weights will acquire the given \( h_{\text{Heb}} \) level after the presentation of a given Hebbian pattern. Consequent Hebbian learning iterations do not reverse the Hebbian weights that already changed through learning and they remain at level \( h_{\text{Heb}} \), whether or not habituation has been affected. Connections to G nodes are not affected by Hebbian learning.

Depending on the implementation of the habituation and overall normalization, the learned
weights do change with respect to the set value of $h_{Heb}$. To make learning efficient, the effect of such a shift in the Hebbian weights should be limited. This, however, can easily lead to the problem of

\[
\text{IF } P_M(i) > (1 + K) \times P_M \text{ AND } P_M(j) > (1 + K) \times P_M \text{ THEN }
\]

\[W'_{M(i) \rightarrow M(j)} = h_{Heb} \]

\[W'_{M(j) \rightarrow M(i)} = h_{Heb}\]

ELSE Do Not Change.

In the modified Hebbian rule [Freeman et al., 1988], the bias coefficient $K > 0$. If $K$ is large, i.e. it is in the order of the standard deviation of the weight distribution, only a small fraction of the weights will be updated at a given iteration of Hebbian learning. In this way, the saturation of the weight space can be avoided. $K$ turns out to be an important parameter that strongly optimizes the learning and generalization performance of the trained KIII network.

3.3. Building efficient learning paradigms

There are several possible learning strategies depending on the way these learning types are combined. In a previous method Algorithm I [Chang et al., 1998], patterns A and B were learned respectively by habituation and Hebbian association, without continuous habituation. The corresponding learning parameters were $h_{hab-A}$ and $h_{Heb-B}$. Here learning is done by Algorithm II in which Hebbian reinforcement is given for patterns A and B presented sequentially, and habituation is superimposed continuously at each time step increment of 1 ms. A typical parameter of continuous habituation is $h_{hab-c}$ and is 0.9995. If no other learning methods influence a given weight, it decreases at a rate of 0.9995802=0.74, after the 600 ms simulation period of a single pattern. By conducting learning for a typical number of 10 patterns, the unreinforced weights diminish asymptotically toward zero. At the end of learning, the weights are fixed in order to perform classification using the trained system. Continuous habituation can be maintained during testing as well, as appears to be the case in biological systems [Freeman, 1975]. For the sake of simplicity, that is not done here.

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<th>Table 1. Parameters of KIII model.</th>
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<td>$W_{P_7\rightarrow P_7}$</td>
</tr>
<tr>
<td>$W_{P_8\rightarrow P_7}$</td>
</tr>
<tr>
<td>$W_{P_7\rightarrow P_8}$</td>
</tr>
<tr>
<td>$W_{P_8\rightarrow P_8}$</td>
</tr>
</tbody>
</table>

The Hebbian learning is characterized by parameters $h_{Heb-A}$ and $h_{Heb-B}$, respectively. Typical values of the used learning parameters are given in Table 2. An advantage of Algorithm II is that
Table 2. Learning parameters.

<table>
<thead>
<tr>
<th>Learning Type</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitation Continuous</td>
<td>h_{hab-c}</td>
<td>0.9995</td>
</tr>
<tr>
<td>Habituation Pattern A</td>
<td>h_{hab-A}</td>
<td>0.85</td>
</tr>
<tr>
<td>Hebbian Pattern A</td>
<td>h_{Heb-A}</td>
<td>0.0159</td>
</tr>
<tr>
<td>Hebbian Pattern B</td>
<td>h_{Heb-B}</td>
<td>0.0095</td>
</tr>
</tbody>
</table>

it is easily generalized to classification problems involving more than two classes. Also, this algorithm gives good results in few learning trials. Due to these advantages, Algorithm II was used in the practical applications described in the present report. Further description of normalization as a form of learning is given in the Appendix.

4. Operation of the Dynamical Memory KIII Network

There are a number of criteria to satisfy in order to achieve a biologically plausible model of chaotic memories. Using the example of the olfaction, these conditions are formulated in the following form based on decades-long studies on the mammalian olfactory system [Freeman et al., 1997; Freeman, 2000a]:

- broad band aperiodic fluctuations in the basal state of the M and G nodes in the OB with $1/f^2$ power spectral densities;
- spatially coherent gamma oscillation in the OB layer;
- temporal fluctuation of pulse and wave densities with amplitude distributions close to normal in the basal state;
- distribution of the pulse densities of the PG nodes concentrated to the right of the maximum gradient of the asymmetric sigmoid function;
- distributions of the pulse densities of the M and G nodes concentrated to the left of the maximum derivative of the sigmoid function;
- mean amplitude of pulse densities of the M nodes at or near zero;
- state transition occurring swiftly, < 20 ms after the onset of each stimulus.

The KIII model satisfies these conditions, so it can be considered as a biologically realistic model in simulations of EEG signals [Kozma & Freeman, 2000]. The procedure for operation of the KIII-based chaotic memory can be summarized as follows:

- Initialize the system according to the procedure described above. This involves both connection weights and nodal potentials.
- Teach KIII to formulate the given input–output relationships using learning Algorithm I or II.
- Optimize the performance using the following control parameters: input noise magnitude $\sigma_{RI}$, Hebbian bias $K$, central noise magnitude $\sigma_C$ and bias $b_C$.

In the present studies, the parameters of the central noise to the AON were fixed at values $\sigma_C = 0.005$ and $b_C = 0.2 \cdot \sigma_C$. The optimization of system performance was conducted using the input noise magnitude $\sigma_{RI}$ and Hebbian bias $K$.

4.1. Demonstration of classification of abstract data set with additive noise

The feasibility of using the KIII network to preprocess input vectors by mapping them to spatial AM of aperiodic oscillations is illustrated in the following example. Preprocessing here means replacing one set of numbers that represents a sensory event with another set of numbers that represents the class to which the event belongs. Each layer of KIII has 64 nodes. Parameters of the basal state have been set to satisfy the requirements of biological plausibility as described in the previous sections. In the first example two classes of artificially generated input data are used. Each datum is a 1 x 64 column vector that is obtained as a superposition of a binary sequence of 0 and 1 and added Gaussian noise. The prototype of class A has ones at positions 1 to 22 and zeros over the rest: 23 to 64. Class B prototype has ones from position 23 to 44 and zeros at the other locations [Kozma & Freeman, 2000]. With a random flip–flop technique over the binary digits, a set of degraded examples has been produced from the prototypes of each class. In the following discussions, three levels of degradation are used:

1. without degradation, i.e. keeping the original prototypes;
2. with 10% degradation of the prototypes;
3. with complete degradation of the prototypes.

Without degradation (Case 1) the classification problem is easy, while the classification is at chance
level with complete degradation (Case 3). Case 2 provides an intermediate level of difficulty. The KIII network is trained with five examples of both classes using Algorithm I in which items in class A are to be accepted, and items in class B are to be rejected as unacceptable or “not A”. The performance of the trained network is tested with five new examples from both classes. A series of 15 tests is conducted with an increasing level of additive noise, up to 150%. The KIII network correctly classifies the test data up to noise levels above 100% in Cases 1 and 2. The classification remains at the chance level in Case 3.

The statistical significance of the classification performance is analyzed using Student’s t-test. The distance between a given test pattern and the centers of pattern clusters A and B is calculated. The null hypothesis states that the distance of the test pattern is the same to both clusters. The alternative hypothesis H1 is that the test pattern is closer to the correct cluster, i.e. the cluster to which it belongs. The significance of H0 is shown in Fig. 3 as the function of the noise-to-signal ratio. Dash, solid, and dash-dot lines stand for cases without degradation, intermediate degradation and complete degradation (chance level). In Case 3 with complete degradation, H0 cannot be rejected. In Case 1, H0 is rejected for all applied additive noise levels. In Case 2 with intermediate complexity, H0 is rejected over an additive noise level of 30% to 70%, showing that noise improves the classification performance of KIII with degraded data over a given range of noise.

4.2. Classification of industrial manufacturing data sets

A standard test bed of four industrial data sets of increasing difficulty was used previously to evaluate the performance of the KIII-based classification method [Yao et al., 1991; Chang et al., 1998]. Each data set was derived by passing 100 small machine parts through a field of ultrasound at 40 KHz and measuring the phase of the reflected sound wave at 64 points [Buckley & Stelson, 1978]. Half of the objects were acceptable a priori, and the other half were defective and unacceptable. The most difficult problem, which was quite intractable, was the classification of Phillips screwdriver tips, half of which were either the wrong size or had missing blades, because the hexagonal shafts created multiple interspersed centroids for each class.

The KIII classification method using Algorithm II for the these data achieves perfect classification after fine tuning of the parameters of the learning rules. The additive input noise level is 28%, while the offset of the Hebbian learning K is zero. The results of comparison of classification with and without KIII processing are summarized in Table 3. Data patterns 1–5 belong to Class A, while patterns 6–10 are from Class B. The distances to the corresponding cluster centers of A and B are indicated in both cases. Table 3 gives the confidence of the classification using Student’s t-test. The null hypothesis H0 holds that cluster A and cluster B are indistinguishable. Hypothesis H1 holds that the items in class A were closer to the cluster centroid A and those in class B were closer to centroid B. The confidence values given in Table 3 are for testing H1 with the KIII network. H0 is rejected with p < 0.0001. In contrast, H0 cannot be rejected in the absence of KIII.

4.3. Identification of EEG stimulus-induced patterns in the rabbit’s visual cortex

A major test of the KIII network is to apply it to the problem of classification of spatial AM patterns of aperiodic EEG oscillations in the gamma range
Table 3. Classification results of bolt apex data.

<table>
<thead>
<tr>
<th>Patterns</th>
<th>With KIII Distance to Center A</th>
<th>With KIII Distance to Center B</th>
<th>W/O KIII Distance to Center A</th>
<th>W/O KIII Distance to Center B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.45e-02</td>
<td>5.49e-02</td>
<td>3.41e+00</td>
<td>2.64e+00</td>
</tr>
<tr>
<td>2</td>
<td>5.19e-02</td>
<td>5.31e-02</td>
<td>1.87e+00</td>
<td>4.12e+00</td>
</tr>
<tr>
<td>3</td>
<td>5.27e-02</td>
<td>5.41e-02</td>
<td>1.49e+00</td>
<td>3.78e+00</td>
</tr>
<tr>
<td>4</td>
<td>5.30e-02</td>
<td>5.33e-02</td>
<td>2.40e+00</td>
<td>2.65e+00</td>
</tr>
<tr>
<td>5</td>
<td>5.27e-02</td>
<td>5.44e-02</td>
<td>3.17e+00</td>
<td>2.39e+00</td>
</tr>
<tr>
<td>6</td>
<td>5.42e-02</td>
<td>5.31e-02</td>
<td>3.85e+00</td>
<td>1.67e+00</td>
</tr>
<tr>
<td>7</td>
<td>5.21e-02</td>
<td>5.01e-02</td>
<td>3.54e+00</td>
<td>1.78e+00</td>
</tr>
<tr>
<td>8</td>
<td>5.53e-02</td>
<td>5.19e-02</td>
<td>4.46e+00</td>
<td>2.20e+00</td>
</tr>
<tr>
<td>9</td>
<td>5.19e-02</td>
<td>5.01e-02</td>
<td>3.47e+00</td>
<td>1.80e+00</td>
</tr>
<tr>
<td>10</td>
<td>5.64e-02</td>
<td>5.38e-02</td>
<td>1.88e+00</td>
<td>3.50e+00</td>
</tr>
</tbody>
</table>

Error Rate: 0% 30%

Significance: $1.937 \times 10^{-4}$  $1.442 \times 10^{-1}$

Fig. 4. Flow diagram of the applied preprocessing methods. Note that data pass through always the same final classifier panel at the bottom of the diagram. Both of the involved preprocessing units, i.e. KIII and the cone-filter, act as feature enhancement tools.
that were recorded from electrode arrays on the OB of rabbits trained under reinforcement to discriminate odorants. These data provided the experimental basis for designing and testing the KIII network. In the meantime the biological paradigm has been extended to the visual, auditory and somatosensory cortices of rabbits, showing that these primary sensory areas likewise reveal spatial AM patterns of gamma oscillations that can be classified with respect to sensory stimuli given in the appropriate modality [Barrie et al., 1996; Ohl et al., 2000]. Two types of AM pattern have been found in neocortex. One type occurs with short latency immediately after stimulus arrival within the first 300 ms. This early AM pattern represents the impact of a discriminated stimulus on the activity of the receiving cortex. The other type occurs with variable latency in the subsequent 500 ms. Its location in time is revealed by a spatial pattern of phase at the center frequency of the gamma oscillation, which has the form of a cone [Freeman & Barric, 2000]. The location of the apex of the cone demarcates the site of nucleation of the state transition by which an AM pattern emerges in the sensory cortex, and the slope of the cone manifests the group velocity by which the transition spreads over the cortical surface. The phase cone therefore serves as a marker by which to locate emergent AM patterns at varying latencies over sequential trials in which either or two discriminated stimuli are presented, one reinforced and the other not.

The data set chosen for a test of the classificatory power of the KIII network was obtained from New Zealand rabbits that were chronically implanted with square arrays (8 x 8) of 64 electrodes surgically implanted on the visual cortex. They were trained to discriminate bright and dim full field flashes [Barrie et al., 1996]. After the rabbits had been trained, 64 electroencephalograms (EEGs) were recorded on each of 20 presentations of the two conditioned stimuli given at the midpoint of each 6 sec trial. The EEGs were digitized at 2 ms intervals, band-pass filtered in the gamma range (20 to 80 Hz), and 64 RMS values were calculated over a 128 ms time window shifted in 32 ms steps. The data for classification were the 40 1 x 64 column vectors at each time step, constituting 30 tests in each set of trials starting from the onset of the stimulus. The 40 data sets were divided into training and test sets of 10 each for the two stimuli. The training sets were used to calculate centroids for each of the two classes. Each test vector specifying an AM pattern expressed a point in 64-space, and the assignment was made by the shortest Euclidean distance to either of the two centroids. Both sets were used in cross-validation. The total goodness of classification was quantified by the % correctly assigned, and by the probability that that number out of the total trials could have occurred by chance. The values from each criterion were plotted as time series.

The results of classification are compared with and without preprocessing using the KIII network. Two data sets are used; see Fig. 4. In one set the time window is stepped at fixed intervals from the stimulus onset, and AM patterns are classified at the same time latency across every trial. In the other set at each time step a search is made for the nearest phase cone before or after that step on every trial, and the AM patterns in those time windows are aligned and compared. The KIII network is trained in Algorithm II, in which habituation is continuous and cumulative over the training set and is turned off during testing. The tuning parameter, K, during the Hebbian learning is set to 0.4-times the SD of the weight distribution. Learning converges in < 10 iterations. Noise is added at incremental levels in repeated tests in order to create

![Graph showing classification performance](https://via.placeholder.com/150)

**Fig. 5.** Classification performance of KIII (solid line segments); the classification performance of the statistical clustering method without KIII is shown by dashed line segments. The EEG patterns are measured in the rabbit visual cortex at time instant 32 ms after the onset of stimulus. Note the significant improvement of the performance of the model at around 40% noise level.
Fig. 6. Temporal evolution of the classification performance of the KIII model during the post-stimulus period in EEG experiments with rabbits. Notations: solid line and star: with KIII; dashed line and cross: without KIII processing. (a) Error rate of EEG classification without cone-filter. (b) Significance of EEG classification without cone-filter. The null hypothesis $H_0$ states that the mean values of the distances to cluster $A$ and cluster $B$ are equal; the alternative hypothesis $H_1$ is that the patterns are closer to their respective clusters $A$ and $B$. (c) Error rate of cone-filtered EEG classification. (d) Significance of EEG classification with cone-filter, with statistically significant rejection of $H_0$ ($p < 0.01$) at 800 ms.
a tuning curve and find the optimal signal-to-noise ratio, see Fig. 5.

The results for the time-locked moving window are summarized in Fig. 6. In Fig. 6(a), the correct classification rate with the KIII network is shown by the solid line and without it by the dashed line. While the classification is evident with and without KIII preprocessing in the first 300 ms after the input stimulus onset, the KII network substantially improves the performance over direct classification [Fig. 6(b)]. Figures 6(c) and 6(d) show the classification rate of AM patterns selected by detection of phase cones. As expected [Ohl et al., 2000] the classification rate is lowered in the first 300 ms, presumably because the AM patterns there appear to be imposed by stimulus input and are not usually emergent with attendant radial phase gradients. There is, however, a statistically significant increase in the classification rate later in the trials, provided that KIII preprocessing is used. The duration of the time window in the trials of significant classification corresponds to the mean duration of phase cones and AM patterns [Freeman & Barrie, 2000], and its location in time near 800 ms precedes the mean behavioral response latency.

5. Discussion

The digital embodiment of the KIII network is designed to simulate the dynamics of the olfactory system, but the two systems differ fundamentally in respect to the mechanisms that generate the noise. The main source in the olfactory system appears to be the synaptic interaction of densely interconnected excitatory neurons, which sustain steady state firing of the neurons, each with an aperiodic pulse train that resembles a Poisson process with a dead time that is imposed by the refractory periods, and that is uncorrelated with the pulse trains of other neurons with which it is interacting. This approximates spatially distributed "white noise", but "colored" by the upper frequency limit imposed by the duration of action potentials (≈ 1 ms) [Freeman, 1996]. The digital noise is provided by a Gaussian random number generator with zero mean and unit standard deviation of the distribution [Freeman et al., 1997; Chang & Freeman, 1998]. In other words, in the olfactory system the noise arises and is sustained by the interaction of myriad microscopic neurons, and the mesoscopic chaos emerges from the constraints imposed by the interaction of each and every neuron in the cooperative mass [Haken, 1983]. In the digital simulation the nodes interact to give deterministic chaos, and the addition of random numbers smooths the attractor landscape. Owing to this distinction the brain activity here called "stochastic chaos" [Freeman, 2000b] can be simulated but not replicated in digital devices, but the possibility of replication with analog devices [Tavares et al., 2000] is open for exploration.

The roles of noise are open to conjecture. On the one hand, the possibility is considered that the role of the additive noise is to compensate for the use of rational numbers instead of real numbers in numerical integration, so that the significance of the noise might be limited to enabling the digital implementation. However, randomization of the terminal bit to induce "terminal chaos" [Zak, 1991] causes only modest delays in the onset of divergence of numerical trajectories [Chang & Freeman, 1998], and randomization of parameter values ("multiplicative noise") with small SDs about their means has negligible effects, so it appears unlikely that the noise serves merely to compensate for the truncation imposed by digital approximation.

On the other hand, there appear to be two biologically significant roles of the types of noise in the simulation. One role is the maintenance of the KII components of the KIII network at nonzero point attractors, as manifested by the observed and simulated oscillatory evoked potentials (impulse responses on electrical stimuli to the brain or Dirac delta function input to the KIII model). Suppression of background activity by anesthetics or by setting feedback gains to zero blocks the oscillations. The other role is stabilization of the chaotic attractor landscape formed by learning, which is manifested in two ways. One is the robustness of attractors and their basins in respect to variations in parameters for gains and amplitudes of inputs. The other is the optimization of pattern classification by adjustment of noise intensities and biases, when state transitions are induced by input [Freeman, 1999].

The improvements in classification resemble the properties of stochastic resonance [Moss & Pei, 1995; Bulsara et al., 1996; Gammatoni et al., 1998], but with important differences [Kozma, 1998; Kozma & Freeman, 1999b]. SR systems have typically a bistable energy function, and there is a weak periodic carrier wave at the input. The additive noise enables the system to surmount the energy
barrier between the stable states and thereby track the periodic input. In "chaotic resonance (CR)" [Brown & Chua, 1999] the energy landscape has multiple basins, and the oscillatory carrier wave is endogenous and aperiodic (approximately $1/f^2$ spectrum). Another difference is that when chaos is introduced in SR the nodes are parameterized to give deterministic chaos (e.g. [Dogaru & Chua, 1998]), whereas in both the olfactory system and the KIII network the chaos emerges as a mesoscopic property. The "CR" effect is apparent in the significant improvement of performance in difficult classifications by modest levels of additive noise given to the input and the AON.

Systematic exploration of the state space of the KII set in the dimensions of the four gain parameters has brought to light the critical role played by the AON in olfactory chaotic dynamics. These four gains list the possible types of direct synaptic action in a mixed population of excitatory and inhibitory neurons, and their strengths relative to each other determine not only the characteristic complex frequency of each KII set, as revealed by the parameters of the measured impulse responses, but also the resting level to which the mixed population returns on relaxation from the impulse perturbation. The basal level of the olfactory bulb at or near zero has been shown by linear analysis to result from pole-zero cancellation of a pole at the origin of the complex plane (a zero eigenvalue in the root matrix, which manifests the essential mechanism of stabilization of the KII set [Freeman, 1975]. The prepyriform cortex (PC) has a negative value for its resting point attractor, as revealed by its silence when it is deprived of tonic input [Freeman & Barrie, 2000].

The anterior olfactory nucleus (AON) has a positive value for its resting point attractor, as a necessary condition for simulation of the aperiodic EEGs of the olfactory system [Freeman, 1987]. Whereas the ranges of acceptable settings for the gain parameters for the OB and PC are broad, the range for the AON is exceedingly narrow. This implies that the AON is a critical node in the olfactory system for its regulation. Several of its properties are consistent with this interpretation. Unlike the OB connections to and from the PC which are broadly divergent-convergent, the AON has topographically organized input and output from and to the OB. The AON has several anatomical components giving both excitatory and inhibitory feedback to the same and contralateral OBs. The AON is also a prime target of axonal projections from other parts of the basal forebrain, which makes it a good candidate for the exercise of control of the olfactory system by other parts of the brain [Kay & Freeman, 1998], and for observer-based delayed feedback control of chaos in the system [Konishi & Kokame, 1998; Liao, 1998].

Future work might usefully be directed toward further studies of the role of the AON in controlling the chaotic dynamics of the olfactory system, both by modeling and by physiological analysis, since this minute structure has been relatively neglected in comparison to numerous studies of the OB, PC and receptor layer. The KII model of the AON might be expanded into a distributed layer, particularly in regard to the possibility of modulation of the KIII attractor landscape by controls equivalent to limbic feedback [Kay & Freeman, 1998], which may provide mechanisms for selective attention through bias controls to enhance detection of faint but important signals, in the manner that animals can be sharply tuned to important odorants, and to sights and sounds as well, considering our evidence that visual, auditory and somatic sensory cortices function with the same basic algorithms as those found in the olfactory bulb [Barrie et al., 1996; Freeman & Barrie, 2000].

Work should be done to expand the KII model of the PC into a distributed layer in order to study its role in readout of bulbar patterns through the divergent-convergent projection of the lateral olfactory tract. This third layer of the central olfactory system may provide the mechanism that detects novel odorants, for which no basin exists, but for which a new basin must be constructed, if it is reinforced, or for which habituation must be updated if it is not reinforced. The olfactory system gives a burst with each inhalation using an attractor that signals "I'm OK" if there is no known odorant in the air stream, but it suppresses a burst in the presence of a novel odor, which indicates the existence of an "I don't know" attractor that gives the chaotic activity necessary to drive Hebbian learning [Skarda & Freeman, 1987].

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Freeman, W. J. & Kozma, R. [2000] “Local-global interactions and the role of mesoscopic (intermediate-


**Appendix A**

**The Olfactory System**

Brains are explored and understood at many hierarchical levels from quantum statistics at synapses to...
human societies in the war on drugs. For biologists the starting level is the cell, which like a person is self-organizing and largely autonomous but cannot survive and function normally without continuous interactions with others of its kind. Neurons comprise a “microscopic” level. They are unique among cells in having long branching filaments that enable each neuron to transmit to $10^4$ others while receiving from $10^4$ others (in order-of-magnitudes). Their outputs are carried by action potentials on axons, which are converted as input at synapses to waves of current in the dendrites of other neurons, but not to itself. The packing density is $\approx 10^6$/mm$^3$, and the range of local connections is about 1 mm, so each neuron contacts only 1% of others within its arbor, and connectivity is dense but sparse. The range of the local interactions that create populations and support spatially coherent activity patterns is about 10 mm including $10^6$--$10^8$ neurons. These populations comprise the “mesoscopic” level. The coupling of multitude of mesoscopic populations by long-distance axonal connections leads to the formation of “macroscopic” systems occupying large parts of the brain. The microscopic activity is observed by recording axonal pulses from single or a few neurons and measured in microns and msec. Mesoscopic activity is recorded in dendritic potentials commonly called EEG waves from many thousand neurons and measured in mm and sec. Macroscopic activity is currently observed indirectly through measurements of blood flow with imaging techniques (fMRI, PET, SPECT, etc.) over many million neurons and is measured in cm and min. The KII model here addresses the mesoscopic level of the olfactory system at the threshold of the macroscopic level.

The olfactory system has four main parts (see Fig. 7): the receptor array (R, $10^8$ in each nostril)

![Fig. 7. Schematic diagram of the olfactory system.](image-url)
in the nose, the olfactory bulb (OB), the anterior olfactory nucleus (AON), and the prepyriform cortex (PPC or PC). Forward transmission from the receptors to the OB is by topographic mapping through the primary olfactory nerve (PON), and from the OB onward through spatial integration by divergent–convergent projection in the lateral olfactory tract (LOT). The AON and PPC send axons back through the medial olfactory tract (MOT) to the OB, but not to the R. There are two main structural classes of neurons: projection neurons with long axons that transmit between layers but also have local side branches called axon collaterals, and interneurons with only local connections. In the olfactory system all projection neurons are excitatory at all of their targets. There are two populations of interneurons in the OB in each hemisphere of the cerebrum. The external interneurons, the periglomerular cells (PG, 10^7), are excitatory to each other (local positive feedback) and feed-forward to the OB projection neurons, the mitral cells (M, 10^5). The internal interneurons, the granule cells (G, 10^7), are inhibitory to each other (also local positive feedback) and to the projection cells (M). Local negative feedback between M and G cells supports the oscillations in the gamma range (20–80 Hz) observed in the bulbar electroencephalogram (EEG). The PG cells provide the regenerative activity which creates the positive bias that is necessary for the oscillations, by maintaining the M and G cells above their thresholds. Excitatory neurons (E, 10^7) in the AON and PPC interact regeneratively with each other, and likewise do inhibitory neurons (I, 10^7) in each part by local positive feedback. Local negative feedback between the projection neurons (E) and interneurons (I) within the AON and within the PPC also provides oscillations in these parts as in the OB. The oscillations are spatially coherent inside each part, owing to the negligibly small local conduction delays, but at three different characteristic frequencies, owing to differences in the three sets of internal feedback gains: positive excitatory feedback, positive inhibitory feedback, and negative feedback, even though the neurons in all three parts have the same open loop time constants (within the limits of the standard errors of measurement).

These three parts of the central olfactory system cannot converge to one frequency, yet are inescapably coupled to each other by long-distance feedback connections, so, as in a ménage trois, they sustain chaos. The broad spectrum activity is facilitated by long-distance feedback delays in the MOT that are greater than the time constants of the neurons in the three parts. Owing to the differences in lengths and conduction velocities of the MOT axons, the delays are distributed and dispersive, so that the effects on transmissions through the several components of the MOT are those of linear low-pass filters. The feedforward delays in the LOT are negligible. Both the dendritic population delays and the axonal long interaction delays are modeled with linear time-invariant second-order ODEs. Static nonlinearities are introduced at the sites of conversion of waves to pulses at the cell bodies where the dendrites and axons contact each other, and time-varying multiplicative parameters are introduced at synapses where incoming pulses are converted to waves of dendritic current.

The role of the 10^6 receptors (R) is transduction of incident chemical odorants into spatial patterns of action potentials. The PON sorts the action potentials in respect to chemical types and maps them onto the OB through the glomeruli, which are 104 nests of synapses in which 10^8 R converge to 10^6 M cells. The PG layer operates on the input by logarithmic range compression, normalization, and spatial coarse-graining. The glomeruli are 0.25 mm in diameter and 0.5 mm center–to-center giving 16 glomeruli/mm^2. Each KO set models the 10^4 M cells or 10^6 G cells in a cluster of glomeruli in 1 mm^2, which is the degree of spatial resolution achieved by recording their EEG activity through each electrode of an array of 64 electrodes (0.5 mm spacing). The OB is the site of learning by habituation, association and normalization, which enable its operations of abstraction and generalization by the formation of AM patterns of oscillation in the EEG gamma range through first order phase transitions. Unlike the PPC the AON has a measurable degree of topographic order in reciprocal mapping with the OB, which may assist in its putative role as a chaotic controller, along with its topological location as the site of central control of the olfactory system by other brain systems. The PPC is the main avenue of read-out, which requires extraction of OB signals and noise reduction ("brain laundry") by the spatiotemporal integral transform of the LOT. The PPC is also engaged in adaptation by learning. Whereas the mechanisms are relatively well understood by which the OB generalizes over its microscopic inputs to form classes relating to experiences with input, the mechanisms by which the PPC generalizes over OB outputs to form classes
relating to decisions following classification remain obscure. These are topics for further study by physiological observation and modeling.

Appendix B
Normalization of Nodal Connectivity

The multilayer KIII model consists of a number of subunits which are KI and KII sets. Examples of these sets are given in Fig. 8. The notations are as follows: P1 and P2 denote the two components of the PG layer. P3 and P4 are the M and C layers. P5 and P6 constitute the AON excitatory and inhibitory layers; P7 and P8 are comparable parts of the PC. Finally, P9 denotes the pyramidal cell node. The connection weights are denoted by lower case letters in Fig. 8. It is possible to write certain balance equations for pairs of the state variables in each component, based on the incoming activation values and the corresponding synaptic weights. The balances in the equations are achieved by solving them to give optimized parameters either analytically or iteratively. The equations are time-varying approximations, as the actual dynamical system has continuously changing variables. Nevertheless, it is meaningful to seek and evaluate these balances, because they represent elemental conditions that should be satisfied in order to obtain a desired system behavior. For example, the exact values of the PG activity levels are unknown. Also, they are spatially distributed, and each spatial node has a different, time-dependent value. Based on neurophysiological considerations [Freeman, 1975, 1987] those values of the state variables should fluctuate around 1.3 to 1.4. With respect to the PG nodes, these balance equations are simple:

\[ P2 = I_6 \ast Q(P1, q_m1) \]  \hspace{1cm} (B.1)

\[ P1 = a_4 \ast Q(P5(1), q_m2) + a_6 \ast Q(P2, q_m1) \]  \hspace{1cm} (B.2)

Here Q is the nonsymmetric sigmoid function of a single parameter q_m, previously introduced.

Fig. 8. Elementary KI and KII sets that are used in the evaluation of homeostatic "local balances" in the KIII system, as defined in the Appendix. In the present study, these local balances are used as an initialization method to speed up computation.
[Freeman, 1975, 1987], and the P's are the state variables to be initialized. The values of qm are well documented experimentally: \( q_{m1} = 1.823 \) in the PG layer and \( q_{m2} = 5.0 \) in all other layers [Chang & Freeman, 1998].

The above set of two equations contains a number of unknowns and it is strongly underdetermined. P1 and P2 are the approximate values of the two PG layers, while P5(1) is the activation of the first excitatory node of the AON. The gains \( l_6 \), \( a_4 \) and \( a_6 \) are also unknown. Based on physiological arguments, as mentioned earlier, the initial values P1 \( \approx \) P2 \( \approx \) 1.4 are used to set the AON value as P5(1) \( \approx \) 0.1. Also, the gain parameter \( l_6 \) can vary only in a very narrow range of values close to unity [Freeman, 1979]. Using the value of \( l_6 = 0.95 \), Eq. (6) is satisfied within an error of 1%. The choice of parameters \( a_4 = 1.7 \) and \( a_6 = 0.8 \) also yields a good approximation of Eq. (7) within a margin of 1%.

The balance equations corresponding to the other layers can be satisfied in a similar manner. The desired average balance in activation level is specified at each node, and this gives a set of values for the weight coefficients. The chosen initial conditions for the balanced state variables are as follows:

\[
P1 = 1.4, \quad P2 = 1.4, \quad P3 = 0.1, \quad P4 = 0.2, \\
P5(1) = 0.1, \quad P5(2) = 0.4, \quad P6(1) = 0.5, \\
P6(2) = -0.2, \quad P7(1) = -0.2, \quad P7(2) = 1.0, \\
P8(1) = 1.0, \quad P8(2) = -0.8, \quad P9 = 0.5.
\]

It is emphasized that the above parameters are arbitrary to some degree, and they should be considered as the results of a nontrivial initialization method. In prior studies [Chang et al., 1998] system was initialized with random conditions. In that case, however, the desired attractor regime of the system was difficult to find within a reasonable computational time. With the specified activation values, suitable weights can be determined from the balance equations. The weights obtained are given in Table 1. With these values, the system can be initialized to generate activation patterns that belong to a high-dimensional aperiodic attractor. The weights are modified during the learning phase, and it is important to keep track of the changes and modify the weights in order to maintain the local balances that assure the desired global dynamical behavior.

In the present implementation, only weights related to the mitral and granule layer are modified during habituation and Hebbian learning, enabling concentration on the balances in these layers. The initial setting provides a stable operation of the KHI model and the system keeps its balances within the desired limits in most cases without further modifications. It is foreseen that in future larger KHI networks the cumulative changes in weights with association and habituation might destabilize the system. Therefore, this locally homeostatic mechanism can be regarded as a third type of learning needed to guarantee long-term global stability, corresponding to well-established control principles in physiology [Freeman, 1975, 2000a].