BIOLOGICALLY MODELED NOISE STABILIZING NEURODYNAMICS FOR PATTERN RECOGNITION

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We present a distributed KIIII model for the olfactory neural system. Low-level Gaussian noise is introduced to the receptors and anterior olfactory nucleus, which biologically models the peripheral and central sources of noise. The additive noise numerically makes the model stable and robust in respect to repeated input-induced state transitions, while improving the simulations of EEG potentials and multiunit activity from the olfactory system. This hybrid dynamics generates a 1/f aperiodic state, which provides an unpatterned basal state for every module to stay in while there is no significant stimulus. Any external input may guide the system to a certain patterned state. The mechanism is fast, fully parallel, under modulatory control, and flexible in absorbing new patterns from unpredictable environments.

1. Introduction

Experimental observations reveal that the electroencephalograms (EEG’s) of the primary sensory cortices for vision, audition, olfaction and touch have spatial patterns relating to external stimuli. A test stimulus can be classified by its correspondence to one of the patterns, which were formed during learning experiences with a set of training stimuli [Freeman, 1987, 1992, 1995; Freeman et al., 1996]. In the language of dynamics, during a normal operation of perception, the sensory cortex maintains a low-dimensional global aperiodic attractor with multiple “wings”. Through a confinement of the cortex into an appropriate “wing” by a particular stimulus, an act of classification occurs. The process can be modeled by a stimulus-induced state transition in a complex dynamical system, which is composed of an array of coupled nonlinear oscillators based in excitatory and inhibitory neuron ensembles, and which is driven by random inputs based in peripheral and central sources of noise. Our purpose here is to use nonlinear stochastic dynamics in fast and correct pattern recognition in accordance with the capabilities of biological intelligence.

Our approach to this problem is first to faithfully model the olfactory neural system and simulate its outputs. We construct a set of parametric ODE’s, which incorporate the anatomical synaptic connections and physiological interactions among cell ensembles. However, the high-dimensional ODE set in expressing chaotic-like behavior [Freeman, 1987; Chang et al., 1998] reveals attractor crowding phenomena [Wiesenfeld & Hadley, 1989; Tsang & Wiesenfeld, 1990]. With any more than a few coupled oscillators, any exceedingly small numerical perturbations in parameters and variables can change the state of the whole system from one global attractor to another. Moreover, if each point on a noisy trajectory is close to a corresponding point of the true trajectory, the noisy trajectory is

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shadowed by the true trajectory [Hammel & Yorke, 1987]. Since the neural system and its computer model both have Lyapunov Exponents that fluctuate above and below zero, we cannot solve the shadowing problem, owing to the resulting numerical instabilities [Dawson et al., 1993, Chang et al., 1998; Freeman et al., 1995].

Stabilization by noise has been reported by others. Fryska and Zohdy put a Markovian process on every ODE of a three-dimensional piecewise linear system to randomize the normalizing-rounding process in floating point arithmetic in numerical calculations [Fryska & Zohdy; 1992]. The shadowing lemma was then valid to assure the existence of a numerically shadowed trajectory. The computer simulations supported the lemma and confirmed the prediction. The authors also introduced Gaussian temporal noise to the KIII ODE’s in order to stabilize the model [Chang et al., 1998], although the shadowing lemma was not valid for a high-dimensional system. The shadowing time could be extended, but the control of the injection noise becomes another critical issue. Likewise, Liljenström and Wu [1995] applied a similar procedure to an associative neural network and claimed that activity, expressed as temporal noise in the model, reduced recall time in associative memory tasks.

In this study, we consider the noise injection from a neurobiological viewpoint. We introduce low-level Gaussian noise at two significant points, the receptors (R) and anterior olfactory nucleus (AON) to simulate both peripheral and central sources of noise in the olfactory system. It offers a solution to stabilize the digital instabilities due to numerically simulating the large-scale neural network.

We then apply the parameter optimization algorithms, which were derived in [Chang & Freeman, 1996; Chang et al., 1998], to perform self-adjustment of parameter values in an acceptable searching domain. Using biological data as the criteria for optimization, we optimize the parameters of the hybrid KIII model to simulate the olfactory system output for specified input. Both the mathematical model and its optimization algorithm, based on the attendant activity perturbation dynamics, should be regarded as integral components of a realistic neural model.

After determining the parameter values, we are able to demonstrate the property of the biological olfactory system in the hybrid KIII model. Start-up period, we generate a 1/f basal state of the granule cell ensemble (G2 node). Then given an external stimulus, the bursts are induced in the olfactory bulb (OB) trajectories at the stimulated channels. The system goes back to the same basal state after the stimulus is ended, as the cortex returns to the previously defined 1/f attractor after the termination of a stimulus, as found in EEG data [Freeman, 1987, 1992]. We thus propose that 1/f aperiodic oscillation provides for a living organ a broad-spectrum state that is unpatterned but ready to be patterned by a small external input.

We next explore the capability to perform pattern classification. The binary input vectors are stored as templates by using a modified Hebbian (an associative correlation) learning rule to adjust the connection strengths among the M1 cell ensembles. Let the model store multiple template patterns simultaneously. Given a test pattern to the receptors, we can identify which class of the templates the alien belongs to, or neither of them, from reading the output bursts of the OB array. Instead of using DC offset to distinguish stimulated patterns [Yao & Freeman, 1990], biologically modeled noise injection enables the KIII to respond to an external distributed stimulus by bursts at the corresponding channels, which performs biological pattern recognition. A related problem, pattern retrieval is also achieved. Given incomplete binary information of the stored pattern, we can recall the complete information.

In Sec. 2, based on the architecture of the olfactory system, the hybrid KIII is formulated. Sections 3–5 numerically analyze the KIII with an injection of biologically modeled noise, which includes three categories of demonstrations: (1) generation of 1/f EEG-like waves, (2) analysis of robustness and stability, and (3) capacity of pattern classification and completion. Parameter values must be biologically acceptable and simulation results must match experimental observations (from EEG data analysis).

2. The Olfactory Neural System and the Hybrid KIII Model

The central olfactory system consists of the bulb (OB), anterior nucleus (AON), and prepyriform cortex (PC). An input from receptors (R) goes to periglomerular (P) and mitral cells (M). The mitral cells transmit to granule cells (G) and to AON
the external capsule (EC) from deep pyramidal cells (C), as well as back to the OB and AON [Freeman, 1975, 1987, 1992]. Figure 1 shows the network topological diagram, called the KIII set.

A population of interaction neurons forms a cell ensemble, which is the basic module of the entire system. When the neurons in an ensemble have no interactions with others, we represent it with a KO set. Both sets of R and C are examples of the KO sets. When the neurons in an ensemble interact reciprocally, we use a KI set, which we model with two excitatory (inhibitory) KO sets mutually connected to form a KI(c) (KI(i)) set. The sets of P and M are examples of the KI(c) sets, and the set of G is the KI(i) set. The reciprocally coupled KI(c) and KI(i) sets form a KII set, which suffices to describe the dynamics of every channel of OB, AON, and PC. From the schematic diagram of principal types of neurons, pathways, and synaptic connections in the olfactory mucosa, bulb, and cortex [Freeman, 1974], the coupling of these KO's, KI's, and KII's by feedforward and time-delayed feedback loops, which are either excitatory or inhibitory, forms a five-layer KIII set to model the entire olfactory system [Freeman, 1987; Yao & Freeman, 1990; Chang et al. 1998]. Based on the knowledge of anatomic

Fig. 1. The topological diagram for the olfactory system, which specifies the connections, external inputs, and also central and peripheral sources of noise for the KIII model.
picture of the olfactory system, we formulate the massively n-channel parallel distributed architecture below.

2.1. Noise, state variables and system parameters

Let \( |N_i^{(p)}(t)| \) and \( N_i^{(c)}(t) \) represent the peripheral and central random noise, respectively. Biologically, the peripheral receptor input noise must be excitatory. Its time series is given by full-wave rectified random numbers in a Gaussian distribution with zero mean and standard deviation (SD) as a parameter. The AON is a site of convergence of multiple controls on the olfactory system from other parts of the brain. The centrifugal noise from the AON is by a spatially coherent divergent projection. Its time series is a Gaussian distribution with positive mean and SD as parameters in the model.

\[
(R_1(t), \ldots, R_n(t))
\]

is the output pulse density activity vector of the sensory receptors.

\[
(P_1(t), \ldots, P_n(t), M_1(t), \ldots, M_{1n}(t), M_{21}(t), \ldots,
M_{2n}(t), G_{11}(t), \ldots, G_{1n}(t), G_{21}(t), \ldots,
G_{2n}(t), E_1(t), E_2(t), I_1(t), I_2(t),
A_1(t), A_2(t), B_1(t), B_2(t))
\]

(1)

denotes the vector of wave activity variables.

\[
(D_1(t), D_2(t), D_3(t), D_4(t))
\]

is the pulse density activity vector of the variables \( E_1(t), E_2(t), A_1(t) \) and \( C(t) \) after four different long delay feedback loops respectively.

\[
(w_{(PL)}, w_{(PD_2)}, w_{(M_1 P)}, w_{(M_1 M_1 L)_{12}}, w_{(M_1 M_1 L)_{13}}, \ldots,
w_{(M_1 M_1 L)_{17}}, w_{(M_1 M_1 L)_{21}}, w_{(M_1 M_1 L)_{23}}, \ldots,
w_{(M_1 M_1 L)_{2n}}, \ldots, w_{(M_1 M_1 L)_{n1}}, w_{(M_1 M_1 L)_{n2}}, \ldots,
w_{(M_1 M_1 L)_{nn}}, \ldots, w_{(M M)}, -w_{(M G)}, w_{(M G)},
-w_{(GG)} - w_{(G_1 G_1 L)}, w_{(G_1 D_1)}, w_{(G_1 D_4)},
w_{(E_1 M_1)}, w_{(E E)}, -w_{(E I)}, w_{(I E)},
-w_{(I I)}, w_{(I D_3)}, w_{(A_1 M_1)}, w_{(A A)},
-w_{(A B)}, w_{(B A)}, -w_{(B B)}, w_{(B C)}, -w_{(C B)})
\]

is the vector of gains. For example, \( w_{(M_1 P)} \) means the connection strength from \( P \) to \( M_1 \), and \( w_{(M_1 M_1 L)} \) is the connection strength between any two \( M_1 \)'s. "+" and "−" indicate mutually excitatory and inhibitory, respectively. Similarly, the vector \( (k_{(PR)}, k_{(M_1 R)}) \) represents the output amplifications from \( R \) to \( P \) and \( M_1 \), respectively. The parameter \( a = 0.220 \text{msec} \) reflects the slower real rate constant of input activity, and \( b = 0.720 \text{msec} \) is the faster real rate constant. For convenience, all the parameters are denoted by vector \( \phi \).

2.2. External inputs

Like other olfactory neurons, each individual receptor has the thresholding property [Freeman, 1975, 1995]. Statistically, an ensemble of the cells results in a sigmoidal-type I/O transformation (similar to those shown in Fig. 2). Assume that there is an external unimodal-shape input to the receptor ensemble, say from \( t = t^{(on)} \) to \( t = t^{(off)} \), at channel \( i \). Then the output pulse density activity \( O_i(t) \) due to the stimulus can be simulated by

\[
O_i(t) = \begin{cases} 
0, & \text{if } t < t^{(on)} \\
I^{(max)} \sin(\omega(t - t^{(on)})), & \text{if } t^{(on)} \leq t < t^{(on)} + \frac{\pi}{2\omega} \\
I^{(max)}, & \text{if } t^{(on)} + \frac{\pi}{2\omega} \leq t < t^{(on)} + \frac{\pi}{2\omega} + t^{(step)} \\
I^{(max)} \sin(\omega(t - t^{(on)} - t^{(step)})), & \text{if } t^{(on)} + \frac{\pi}{2\omega} + t^{(step)} \leq t < t^{(on)} + \frac{\pi}{\omega} + t^{(step)} \\
0, & \text{if } t^{(on)} + \frac{\pi}{\omega} + t^{(step)} \leq t
\end{cases}
\]

and \( t^{(off)} = t^{(on)} + \frac{\pi}{\omega} + t^{(step)} \). The parameter \( \omega \) decides the stimulus rising and decaying rate, and \( t^{(step)} \) controls the time duration of the stimulus.

The receptor ensemble due to a unimodal-shape input.

It is known that the receptors are not synaptically interconnected, and thus do not form co-
property, the total output pulse density activity of the individual receptor ensemble at channel $i$ is truncated as,

$$R_i(t) = \begin{cases} O_i(t) + |N_{i}^{(p)}(t)|, & O_i(t) + |N_{i}^{(p)}(t)| < I^{(\text{max})}, \\ I^{(\text{max})}, & \text{otherwise}, \end{cases}$$

which is the input to the periglomerular cells and mitral cells [see Fig. 3(b)].

2.3. $I/O$ sigmoidal nonlinearity

Let $x(t)$ denote an element of the wave activity vector, which was derived from the Hodgkin–Huxley equations [Freeman, 1979] and evaluated by experiments [Eckman & Freeman, 1991]. $q^{(LR)}$, defining the least upper bound of $Q(x(t), q^{(LR)})$, is a constant for all the ensembles belonging to one layer ($LR \in \{P, OB, AON, PC, C\}$) [Yao & Freeman, 1990; Chang et al., 1998]. To know the nonlinear...
derivative of $Q(x, q^{(LR)})$ function w.r.t. $x$, 
\[
Q'(x, q^{(LR)}) \triangleq \frac{\partial Q(x, q^{(LR)})}{\partial x} \triangleq \exp \left( x - \frac{1}{q^{(LR)}} [\exp(x) - 1] \right), \tag{3}
\]
which is called gain function (see the bottom plots of Fig. 2).

2.4. Network of ensembles

Following the topological diagram of Fig. 1, we implement the $n$-channel hybrid KII model by the

\[
\frac{1}{B} [\dot{M}_1(t) + A\dot{M}_1(t) + BM_1(t)] = w_{(M_1P)}Q(P_1(t), q^{(P)}) + w_{(MM)}Q(M_2(t), q^{(OB)})
\]
\[
- w_{(MG)}[Q(G_1(t), q^{(OB)}) + Q(G_2(t), q^{(OB)})]
\]
\[
+ \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} w_{(M_1M_1)_{ij}} Q(M_1(t), q^{(OB)})
\]
\[
+ k_{(MG)}R_i(t);
\]

\[
\frac{1}{B} [\dot{M}_2(t) + A\dot{M}_2(t) + BM_2(t)] = w_{(MM)}Q(M_1(t), q^{(OB)}) - w_{(MG)}Q(G_1(t), q^{(OB)});
\]

\[
\frac{1}{B} [\dot{G}_1(t) + A\dot{G}_1(t) + BG_1(t)] = -w_{(GG)}Q(G_1(t), q^{(OB)})
\]
\[
+ w_{(GM)}[Q(M_1(t), q^{(OB)}) + Q(M_2(t), q^{(OB)})]
\]
\[
- \frac{w_{(G_1G_1)}}{n-1} \sum_{j=1, j \neq i}^{n} Q(G_1_j(t), q^{(OB)})
\]
\[
+ w_{(G,D)} D_1(t) + w_{(G,D)} D_4(t);
\]

\[
\frac{1}{B} [\dot{G}_2(t) + A\dot{G}_2(t) + BG_2(t)] = w_{(GM)}Q(M_1, q^{(OB)}) - w_{(GG)}Q(G_1(t), q^{(OB)});
\]

for the $AON$ layer

\[
\frac{1}{B} [\dot{E}_1(t) + A\dot{E}_1(t) + BE_1(t)] = w_{(EE)}Q(E_1(t), q^{(AON)})
\]
\[
- w_{(EI)}[Q(I_1(t), q^{(AON)}) + Q(I_2(t), q^{(AON)})]
\]
\[
+ \frac{w_{(E_1M_1)}}{n} \sum_{j=1}^{n} Q(M_1_j(t), q^{(OB)}) + N^c(t);
\]

\[
\frac{1}{B} [\dot{E}_2(t) + A\dot{E}_2(t) + BE_2(t)] = w_{(EE)}Q(E_1(t), q^{(OB)}) - w_{(EI)}Q(I_1(t), q^{(AON)});
\]

\[
\frac{1}{B} [\dot{I}_1(t) + A\dot{I}_1(t) + BI_1(t)] = -w_{II}Q(I_2(t), q^{(AON)}) + w_{(IE)}[Q(E_1(t), q^{(AON)})
\]
\[
+ Q(E_2(t), q^{(AON)})] + w_{(I,D)} D_3(t);
\]

\[
\frac{1}{B} [\dot{I}_2(t) + A\dot{I}_2(t) + BI_2(t)] = -w_{II}Q(I_1(t), q^{(AON)}) - w_{(E,D)} D_3(t);
\]

where $Q(x, q^{(LR)})$ is the ODE set. We shorthand denote it by $\dot{x}(t) = F(x(t), p)$. A single (double) dot over a variable means the 1st (2nd) derivative of the variable w.r.t. time. For the $P$ layer

\[
\frac{1}{B} [\dot{P}_1(t) + A\dot{P}_1(t) + BP_1(t)]
\]
\[
= \frac{w_{(PPL)}}{n-1} \sum_{j=1, j \neq i}^{n} Q(P_j(t), q^{(P)})
\]
\[
+ w_{(PD_2)} D_3(t) + k_{(PR)} R_i(t);
\]

for the $OB$ layer

\[
\frac{1}{B} [\dot{G}_1(t) + A\dot{G}_1(t) + BG_1(t)]
\]
\[
= -w_{(GG)}Q(G_1(t), q^{(OB)})
\]
\[
+ w_{(GM)}[Q(M_1(t), q^{(OB)}) + Q(M_2(t), q^{(OB)})]
\]
\[
- \frac{w_{(G_1G_1)}}{n-1} \sum_{j=1, j \neq i}^{n} Q(G_1_j(t), q^{(OB)})
\]
\[
+ w_{(G,D)} D_1(t) + w_{(G,D)} D_4(t);
\]

for the $AON$ layer

\[
\frac{1}{B} [\dot{E}_1(t) + A\dot{E}_1(t) + BE_1(t)] = w_{(EE)}Q(E_1(t), q^{(AON)})
\]
\[
- w_{(EI)}[Q(I_1(t), q^{(AON)}) + Q(I_2(t), q^{(AON)})]
\]
\[
+ \frac{w_{(E_1M_1)}}{n} \sum_{j=1}^{n} Q(M_1_j(t), q^{(OB)}) + N^c(t);
\]

\[
\frac{1}{B} [\dot{E}_2(t) + A\dot{E}_2(t) + BE_2(t)] = w_{(EE)}Q(E_1(t), q^{(OB)}) - w_{(EI)}Q(I_1(t), q^{(AON)});
\]

\[
\frac{1}{B} [\dot{I}_1(t) + A\dot{I}_1(t) + BI_1(t)] = -w_{II}Q(I_2(t), q^{(AON)}) + w_{(IE)}[Q(E_1(t), q^{(AON)})
\]
\[
+ Q(E_2(t), q^{(AON)})] + w_{(I,D)} D_3(t);
\]

\[
\frac{1}{B} [\dot{I}_2(t) + A\dot{I}_2(t) + BI_2(t)] = -w_{II}Q(I_1(t), q^{(AON)}) - w_{(E,D)} D_3(t);
\]
for the $PC$ layer

$$
\begin{align*}
\frac{1}{B} [\dot{A}_1(t) + AA_1(t) + BA_1(t)] &= w_{(AA)} Q(A_2(t), q^{(PC)}) - w_{(AB)}[Q(B_1(t), q^{(PC)}) \\
&+ Q(B_2(t), q^{(PC)})] + \frac{w_{(A_1M_1)}}{n} \sum_{j=1}^{n} Q(M_1(t), q^{(OB)}) \\
\frac{1}{B} [\dot{A}_2(t) + AA_2(t) + BA_2(t)] &= w_{(AA)} Q(A_1(t), q^{(PC)}) + w_{(AB)} Q(B_1(t), q^{(PC)}) \\
\frac{1}{B} [\dot{B}_1(t) + A\dot{B}_1(t) + BB_1(t)] &= -w_{(BB)} Q(B_2(t), q^{(PC)}) + w_{(BA)} [Q(A_1(t), q^{(PC)}) \\
&+ Q(A_2(t), q^{(PC)})] + w_{(B_1C)} Q(C(t), q^{(C)}) \\
\frac{1}{B} [\dot{B}_2(t) + A\dot{B}_2(t) + BB_2(t)] &= w_{(BA)} Q(A_1(t), q^{(PC)}) - w_{(BB)} Q(B_1(t), q^{(PC)})
\end{align*}
$$

for the $C$ layer

$$
\frac{1}{B} [\dot{C}(t) + AC(t) + BC(t)] = -w_{(CB)} Q(B_1(t), q^{(C)}).
$$

For the $DL$ nodes

$$
T^{(s)}_t T^{(e)}_t \left[ \ddot{D}_t(t) + \left( \frac{1}{T^{(s)}_t} + \frac{1}{T^{(e)}_t} \right) \dot{D}_t(t) + \frac{1}{T^{(s)}_t n^{(e)}_t} D_t(t) \right] = Q(y_t(t), q^{(LR)}),
$$

with $D_t(0) = \dot{D}_t(0) = 0$ for all $t \in \{1, 2, 3, 4\}$. Here, if $LR$ is $AON$ then $y_t(t)$ is $E_1(t)$ for $t \in \{1, 2\}$; if $LR$ is $PC$ then $y_3(t)$ is $A_1(t)$; if $LR$ is $C$ then $y_4(t)$ is $C(t)$. The rationale and validation for using the 2nd order ODE's to perform the delay in long feedback loops were presented in [Chang et al., 1998], where a set of differential-delay equations for the early KII model was converted into a set of ODE's.

### 2.5. Adaptation of connections

The synapses connecting excitatory neurons at different channels within the $OB$, within the $AON$ and within the $PC$ are inferred to be selectively enhanced during learning in the olfactory system [Freeman, 1975]. Learning does not cause changing at other synapses, particularly at the input to the bulb and cortex. The input synapses of the $OB$, $AON$ and $PC$, are dependently modifiable by volume of input and behavior arousal, but not by habituation and associative learning [Freeman, 1975; Yao & Freeman, 1990; Freeman, 1995]. Because $AON$ and $PC$ layers are not distributed in this study, $w_{(M_1M_1L)}^{(1)}$'s $(i, j \in \{1, 2, \ldots, n\})$ are the only adaptive parameters under the hypothesis.

Here, we continue to use the Hebbian-type input correlation rule, which was developed in [Freeman et al., 1988] for studying the distributed KII bers, $w_{(M_1M_1M_1)}^{(high)} > w_{(M_1M_1M_1)}^{(low)}$. Let $V_\alpha = (V_{\alpha_1}, V_{\alpha_2}, \ldots, V_{\alpha_m})$ with $\alpha \in \{1, 2, \ldots, m\}$ represent $m$ n-valued binary vectors. Going through every $\alpha, i$ and $j$,

$$
\begin{align*}
w_{(M_1M_1M_1)}^{(low)} &= w_{(M_1M_1M_1)}^{(low)} \\
w_{(M_1M_1M_1)}^{(high)} &= w_{(M_1M_1M_1)}^{(high)},
\end{align*}
$$

otherwise.

Each memory, corresponding to the spatial pattern $V_\alpha$, can be retrieved by any inputs sufficient to evoke that pattern.

### 3. Generation of 1/f-type EEG Waves

We study the 64-channel hybrid KII model (i.e. $n = 64$). All the computer runs are done on Power Macintosh machines by using the simple fourth-order Runge–Kutta method to perform ODE integrations (except presentations in Fig. 9). The continuous time equations in Sec. 2 are transformed to corresponding discrete time maps. The time step for numerical integration $\Delta t$ is always set to 1.0 msc. The initial values of variables $[see
zero (except presentations in Fig. 6). The external input is on from 1101 to 1300 msec at Channel 2 only (except presentations in Figs. 10-12).

The parameter optimization algorithms, developed in [Chang & Freeman, 1996; Chang et al., 1998], are applied to optimize parameters to simulate the EEG potentials from the olfactory neural system. Low-level Gaussian noise is injected at two biologically significant points, the receptors ($R$ node) and the anterior olfactory nucleus ($E_1$ node). The stable nonzero point attractor in the $K^{P}$ sets ($PG$) holds the dynamical model in an oscillatory state. The random noise stabilizes the aperiodic attractor of that state. Optimization criteria require

![Graphs showing time series and amplitude histograms](image)

Fig. 4. The values of the KIII's variables and their first derivatives are all zeros at $t = 0$: (a) Time series of $P_1(t), M_1(t), G_2(t)$ (first channel of the OB array), $N^{(d)}(t), E_1(t)$ and $A_1(t)$; (b) Log-log plots of power spectra from 301 to 2100 msec; (c) Amplitude histograms of the time series from 301 to 2100 msec. (The dotted lines are the corresponding gain functions, which are used to show a normal operation of PG and OB cells. The means and standard deviations of the outputs of $P_1(t), M_1(t), G_2(t), E_1(t)$ and $A_1(t)$ are 1.361, -0.010, -0.410, 0.041, -0.003 and 0.092, 0.035, 0.189, 0.172, 0.017 respectively.)
Fig. 4. (Continued)
that the hybrid KIPI's outputs have:

1. aperiodic oscillations with near $1/f$-type power spectra, which are stable and robust in the sense that statistical measures of the trajectories are "convergent" under perturbations of either initial conditions of variables or parameters (including random number generation seed);

2. nearly periodic oscillations with peaks in the gamma range during excitatory inputs corresponding to EEG bursts on inhalation;

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Fig. 4. (Continued)

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Fig. 5. (a) Time series of $P_2(t)$, $M_{12}(t)$ and $G_{22}$ (second channel of the OB array). (b) Log-log plot of power spectrum and amplitude histogram (Mean = -0.386 of Standard Deviation = 0.217) of the $G_{22}(t)$ before stimulus (from 301 to 1100 msec). (c) Log-log plot of power spectrum and amplitude histogram (Mean = -0.834 of Standard Deviation = 0.558) of $G_{22}(t)$ during stimulus (from 1301 to 2100 msec). (d) Log-log plot of power spectrum and amplitude histogram (Mean = -0.193 of Standard Deviation = 0.348).
3. spatially coherent wave forms over all modules of the olfactory bulb (OB) both in and between bursts;
4. transition from background to burst less than 10 msec with the stated input;
5. amplitude histograms in both the wave and pulse modes that are nearly Gaussian;
6. periglomerular cell (PG) ensembles operating above the point of maximal slope of the sigmoidal function and well below the maximal (ex-

7. OB ensembles operating below the point of maximal slope of the sigmoidal function and well above the minimal (inhibitory) asymptote (see the top right of Fig. 4);
8. nearly equal means for the $G_1$ and $G_2$ nodes; and
9. near zero offset of the mitral cell ensembles ($M_1$ nodes) irrespective of the mean amplitudes of the $G_1$ and $G_2$ nodes.

Then we have the optimized parameter val-
are obtained from fitting experimental impulse response in an open loop state [Freeman, 1964]. For $n$-channel $\text{KI}^{(P)}$'s, $q^{(P)} = 1.824$, $w_{(PPL)} = 0.900$, and $k_{(PR)} = 0.500$; for $n$-channel $\text{KI}^{(OB)}$'s, $q^{(OB)} = 5.000$, $w_{(MM)} = 1.500$, $w_{(MG)} = 2.063$, $w_{(GM)} = 2.323$, $w_{(GG)} = 2.445$, and $k_{(MR)} = 1.000$; for $\text{KI}^{(AGN)}$, $q^{(AGN)} = 5.000$, $w_{(EE)} = 1.002$, $w_{(ED)} = 1.426$, $w_{(EB)} = 1.372$ and $w_{(II)} = 1.571$; for $\text{KI}^{(PC)}$, $q^{(PC)} = 5.000$, $w_{(AA)} = 0.823$, $w_{(AB)} = 1.938$, $w_{(BA)} = 1.947$ and $w_{(BB)} = -2.354$; for $\text{KO}^{(C)}$, $q^{(C)} = 5.000$. The interconnection parameters between those subsets are $w_{(M_1P)} = 0.050$, $w_{(M_1M_1L)} = 0.599$, $w_{(M_1M_1)} = 2.100$, $w_{(E_1M_1)} = 1.311$, $w_{(A_1M_1)} = 1.710$, $w_{(G_1G_1L)} = 0.580$, $w_{(CB_1)} = 1.543$, $w_{(B_1C)} = 0.698$, and $w_{(G_1D_1)} = 2.349$, $w_{(PD_2)} = 1.087$, $w_{(D_1D_2)} = 2.553$, $w_{(G_1D_4)} = 2.305$, and $T_1^{(e)} = 20.000$, $T_1^{(e)} = 11.000$, $T_2^{(s)} = 22.000$, $T_2^{(e)} = 15.000$, $T_3^{(s)} = 21.000$, $T_3^{(e)} = 12.000$, $T_4^{(s)} = 30.000$, $T_4^{(e)} = 24.000$. The external

![Fig. 6. The values of the KIII's variables and their first derivatives at $t = 0$ are all random numbers, generated by a Gaussian distribution with zero mean and standard deviation $= 0.050$: Time series of $P_1(t)$, $P_2(t)$, $M_{11}(t)$, $M_{12}(t)$, $G_{21}(t)$ and $G_{22}(t)$. The light lines, which are the $P_1(t)$, $P_2(t)$, $M_{11}(t)$, $M_{12}(t)$, $G_{21}(t)$ and $G_{22}(t)$ trajectories shown in Figs. 4 and 5, are put down to compare with the outputs from different initial conditions. The means and standard deviations of the outputs (bold]
input parameters are $I^{(\text{max})} = 0.850$, $\omega = 0.020\pi$, $t^{(on)} = 301,000$, and $t^{(step)} = 150,000$ for each stimulated channel. The details about how to determine these values were presented in [Freeman, 1975; Freeman & Shimoide, 1994; Chang & Freeman, 1996; Chang et al., 1998]. Here, $N_i^{(p)}(t)$ is generated by an independent identical Gaussian distribution with a zero mean and standard deviation = 0.070; $N^{(c)}(t)$ is Gaussian distributed with positive mean = 0.200 and standard deviation = 0.700.

Figure 4 shows the standard optimized output trajectories for a nonstimulus channel. The corresponding output due to stimulus $R_2(t)$ (i.e. time series of Channel 2) is shown in Fig. 5. To shorten the run time, only the first 300 msecs is considered as a warm-up period, and the following 800 msecs give a basal state. The hybrid KIII trajectory then jumps to a near “periodic” state (around 70 Hz) in the next two hundred msecs due to presenting a 200-msec stimulus input at the receptors with intensity 0.850, and goes back to the same basal state, shown by another 800 msecs, after the stimulus is ended.

Clearly, the original deterministic KIII dynamics does contribute to generate an EEG-like wave (a less random state) from the hybrid process. Notice that with window size 2100 points, our numerical simulations also show that the linear correlation

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**Fig. 7.** Same parameter values and initial conditions as used in Figs. 4 and 5 except $k_{(MM)} = 1.800$: Time series of $P_i(t)$, $P_2(t)$, $M_{i1}(t)$, $M_{i2}(t)$, $G_{21}(t)$ and $G_{22}(t)$. (The light lines, which are the $P_i(t)$, $P_2(t)$, $M_{i1}(t)$, $M_{i2}(t)$, $G_{21}(t)$ and $G_{22}(t)$ trajectories shown in Figs. 4 and 5, are put down to compare with the outputs from changing parameter values. The means and standard deviations of the outputs (bold line) of $P_i(t)$, $M_{i1}(t)$ and $G_{21}(t)$ from 301 to 2100 msec are $1.365$, $-0.623$, $
coefficients between \( N^{(c)}(t) \) and \( E_1(t) \) is 0.225, and between \( M_1(t) \) and \( E_1(t) \) is 0.176.

4. Stability and Robustness

The living brain exhibits stable and robust background activity. Certain system perturbations from environment do not destroy or impair its structures and functions. To trust a simulation solution, the trajectory should also be shadowed under numerical round-off/truncation. We would thus like its model KIII to be stable and robust in qualitative dynamics to perturbations in variables and parameters.

However, it seems not possible to solve the neural networks (a high-dimensional deterministic ODE set) for modeling nonrepresentational, non-computational brain dynamics. Consider the size of basin of attraction of dynamics. For an intrinsically high-dimensional nonlinear system \( x(t) = F(x(t), p) \), each \((M_{1i}, M_{2i}, G_{1i}, G_{2i})\) set is an oscillator. The number of attractors increases dramatically as the number of oscillators in the system increases. (The ratio of size of phase space and the number of attractors approaches zero as the dimension approaches infinity.) The size of basin of attraction becomes vanishingly small, and the attractor crowding makes the model sensitive to any extremely low levels of system changes [Wiesenfeld & Hadley,
dimension of dynamics itself. The existence of a Lyapunov exponent fluctuating about zero is expected to be common in simulations of high-dimensional dynamics [Dawson et al., 1994]. When the region around the numerical trajectory develops a second expanding direction while the corresponding Lyapunov exponent stays above zero, it is pushed away from the segment of true

Fig 8. Repeat the simulations in Figs. 4 and 5 with other random number generation seeds: (a) Time series of $P_1(t), P_2(t), M_1(t), M_{12}(t), G_{21}(t)$ and $G_{22}(t)$; (b) Log-log plots of power spectra from 301 to 2100 msec; (c) Amplitude histograms of the time series from 301 to 2100 msec. (The means and standard deviations of the outputs of $P_1(t), M_{11}(t)$ and $G_{21}(t)$ are
Fig. 8. (Continued)

Fig. 9. Repeat the simulations in Fig. 4 by the modified Runge-Kutta integrator: (a) Time series of $P_1(t)$, $M_1(t)$ and $G_2(t)$; (b) Log-log plots of power spectra from 301 to 2100 msec; (c) Amplitude histograms of the time series from 301 to 2100 msec. (The means and standard deviations of the outputs of $P_1(t)$, $M_1(t)$ and $G_2(t)$ are 1.335, -0.0160, -0.277 and 0.0147, 0.0059, 0.021, respectively.)
trajectory and leads to failure of shadowability [Abraham & Smale, 1970]. On the other hand, it is also possible that any tiny perturbation in the computation of \( \mathbf{x}(t) + \mathbf{F}(\mathbf{x}(t), \mathbf{p})\Delta t \) can push \( \mathbf{x}(t + \Delta t) \) across a stable manifold and make the numerical trajectory no longer shadowed by the true one [Dawson et al., 1994].

Our empirical observations on the original KIII show that the difference due to truncation/round-off on terminal bits from changing computer languages, implementation algorithms, or even mathematically equivalent model formulas can propagate out spatio-temporally and cause divergent numerical solutions [Chang et al., 1998]. We agree with the statements put down in [Tsang & Wiesenfeld, 1990; Dawson et al., 1994; Blum, 1991]. Tsang and Wiesenfeld stated: “In large arrays of coupled nonlinear oscillators · · · due to competition between huge numbers of coexisting dynamical states · · · no matter how small the level of random noise, eventually there is an \( N \) large enough that the distance between attractors is smaller than the typical fluctuation due to noise.” Dawson et al. stated: “We believe that in systems with high-dimensional chaos, trajectories with intrinsic noise, such as computer-generated pseudo-trajectories, can be shadowed only for short times.”

Fig. 9. (Continued)

...ical analysis and scientific computation one must extend the new model of computation to incorporate notions of round-off error, condition numbers and approximate solutions.”

Before numerically studying the stability and robustness of the hybrid KIII, i.e. the equation set in Sec. 2, we give their formal definitions as follows. Let \( t_0 \) be the initial time that the oscillator system is modeled, \( \mathbf{x}(t) \) be a trajectory vector of the hybrid KIII, \( \mathbf{x}(\varepsilon)(t) \) be the associative trajectory vector under perturbations of initial conditions of variables \( \mathbf{x}(t_0) \), \( \mathbf{x}(\varepsilon)(t_0) \) be the associative trajectory vector under perturbations of system parameters \( \rho \) (or driving forces (noise) \( \mathbf{R} \), etc.) We say an output trajectory vector is stable if the statistical measures \( \mathcal{M}(\cdot) \) on \( \mathbf{x}(t) \) and \( \mathbf{x}(\varepsilon)(t) \) are close enough. That is, for every \( \varepsilon > 0 \) there exists \( \delta(\varepsilon, t_0) \) such that \( ||\mathcal{M}(\mathbf{x}(t)) - \mathcal{M}(\mathbf{x}(\varepsilon)(t))|| < \varepsilon \) whenever \( ||\mathbf{x}(t_0) - \mathbf{x}(\varepsilon)(t_0)|| < \delta(\varepsilon, t_0) \). Similarly, it is robust, if for every \( \varepsilon > 0 \) there exists \( \delta(\varepsilon) \) such that \( ||\mathcal{M}(\mathbf{x}(t)) - \mathcal{M}(\mathbf{x}(\varepsilon)(t))|| < \varepsilon \) whenever \( ||\mathbf{R}|| < \delta(\varepsilon) \) (or \( ||\Delta \mathbf{R}|| < \delta(\varepsilon) \)). Instead of the classical definition of stability and robustness [Chang & Freeman, 1998], in high-dimensional stochastic neurodynamics they mean “convergence” of statistical measures on output trajectories under perturbations of initial conditions of variables and param-
The additive noise makes the KIII stable and robust in respect to repeated input-induced state transitions, while improving the simulations of EEG and multidimensional activity from the olfactory system. By using the same parameter values as given in Sec. 3, Figs. 6–8 reveal the stability and robustness property of the parameterized 64-channel hybrid KIII, respectively. Obviously, they all have very close statistical properties as the unperturbed one in Figs. 4 and 5. We conclude that brain dynamics is not best simulated in digital computers with point, limit cycle and chaotic attractors in a stationary and autonomous system, but with spatially coherent hybrid states.

Figure 9 is an additional simulation, which shows the closeness (invariance) of statistical measures on the hybrid KIII trajectories under the variations of ODE integrator. We used the modified Runge-Kutta method for ODE's with \( \Delta t = 1.0 \) msec to repeat the simulations shown in Figs. 4 and 5. This method has an adaptive stepsize control, which helps achieve more accuracy. Due to noise injection to the KIII, the adaptive-stepsize-control integrator makes a determination and change of stepsize very frequently, which makes the computer run time relatively long. The resulting gains in precision can increase the computational time over a hundred fold. Because the statistical properties of outputs are not changed by the choice of integrator, we choose the faster one to save run time.

5. Applications

In this section, we explore and apply the capability, that the system can return to the previously defined 1/f attractor after the stimulation of the stimulus, as found from experimental EEG data, to perform biological pattern classification and completion.

5.1. Pattern classification

Consider the analogous procedure as described in [Yao et al., 1991]. We take and preprocess data by:

1. Sensing and digitizing: Take an image of a target object, and represent it as real numbers. Categorize and collect enough data.
2. Preprocessing data: Select only independent data sets to minimize the feature space dimension, say \( n \), by studying correlations between feature data according to its statistical parameters to reduce undue influence.
3. Making templates: Calculate the centroids of all classes. Make every class a binary vector representation according to the centroids by way of statistical approach, geometrical approach, etc.

We apply the hybrid KIII to store templates and classify data by:

1. Storing templates: Use a modified Hebbian (associative correlation) learning rule to adjust the connection strengths among the \( M_1 \) nodes of the hybrid KIII in order to memorize the template patterns.
2. Enhancing feature: Given other \( n \)-valued data, they are binarized one-by-one according to all the centroids calculated in 3. The binary vector indicates the distributed stimulus to the desired channels of the hybrid KIII receptors.
3. Classification: Measure the amplitude of the wave forms of the outputs of the hybrid KIII \( OB \) array of \( n \) modules during the external unimodal-shape input on, determine a point in \( n \)-space and calculate the Euclidean distance of the point to the centroids of the training set output patterns. The minimum distance provides the classification.

Again, the simulations are performed by using the 64-channel hybrid KIII with the same parameter values as in Sec. 3. Equation (4), and two template patterns

\[
V_1 = (1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, \),

\[
0, 0, 0, \ldots, 0)_{64}
\]

and

\[
V_2 = (0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, \),

\[
0, 0, 0, \ldots, 0)_{64}
\]

are used to set up values of \( w_{(Mj, Mj,j)} \)'s. The hybrid KIII is given distributed stimulus to receptors from 1101 msec to 1300 msec according to \( V_1 \) (i.e. \( R(t) \) in Eq. (2) is on at Channels 1, 2, 4, 6, 7, 9, 13, and 15). Figure 10 shows the output of \( G_2 \) node having the burst wave form during the stimulated period at the corresponding channels. Similarly, we can give the distributed stimulus according to \( V_2 \) and obtain the corresponding output pattern. For presentation convenience, we use and analyze only the first 16 channels of the \( OB \) array in the simula-
Fig. 10. Output $G_2(t)$ wave forms from the first 18 channels of the 64-channel hybrid KIII, which show how the pattern $V_1$
Fig. 11. (a) The test pattern $X$ is recognized and classified to $V_1$, which has been memorized by the hybrid KIII. (b) The
Fig. 12. By inputting an incomplete pattern $Z$ to the hybrid KIII, the stored pattern $V$ is completely retrieved.
Channel 16 are all close to each other. As shown by the outputs of $G_{217}(t)$ and $G_{218}(t)$ in Figs. 10–12, there are no bursts generated because of no stimulus input.

We next give a stimulus to channels of the receptors according to any pattern

$$X = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, \ldots, 0)_{64},$$

which is close to $V_1$ compared with $V_2$ in the sense of Hamming Distance. Figure 11(a) shows that the output pattern is classified to $V_1$ by way of Classification Procedure 3. Similarly, given the input pattern

$$Y = (0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \ldots, 0)_{64},$$

which is close to $V_2$, Fig. 11(b) shows that the OB output burst pattern is classified to $V_2$. As we can see, the hybrid KIII model recognizes and classifies any 64-dimensional binary input pattern to the closest memorized pattern. This result supports the hypothesis that the system can perform pattern recognition and classification when new basins of attraction are introduced by Hebbian-type learning.

In principle, the KIII model will classify any input pattern $X$, close to a stored template $V_1$, to pattern $V_1$ by the current learning algorithm and plausible classification measure. However, in reality we may have an actual $V_2$ pattern, which is “taken grant” to be close to $V_1$, is incorrectly classified to pattern $V_1$. The biologically based KIII model should be able to correctly handle such sophisticated situations as a biological brain system can do. More biologically oriented learning rules and classification measures are still under investigation. More realistic industrial data will be applied for further tests and study.

### 5.2. Pattern completion

A related problem, pattern completion is also achieved. We train the hybrid KIII to memorize a pattern

$$V = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1),$$

Then, give its partial information (i.e. incomplete binary vector)

$$Z = (0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0),$$

which is decoded to 1; otherwise it is 0. In this way, the complete information, $V$ is retrieved. Although in the test simulation there are 50% 1's missing, the outputs with and without bursts at the corresponding channels are still clearly present. Hamming Distance can be a measurement. Figure 12 has bursts at Channel 1 to Channel 16 in the OB array, which demonstrates the retrieval capability of the parameterized hybrid KIII.

### 6. Conclusions and Future Work

Point, limit cycle and chaotic attractors exist in large-scale biological neural networks, and they are stable and robust once the steady states occur. When the noise-free KIII model is placed into basins of point and limit cycle attractors, it is stable and robust. However, when the same model is placed into the basins of chaotic attractors, it is supersensitive to any numerical perturbations. Simulating biological source of noise by introducing low-level Gaussian noise to the receptors and to the AON, we make the original chaotic KIII numerically stable and robust. Stability and robustness here mean “convergence” of statistical measures on the KIII’s output trajectories under perturbations of initial conditions of variables and parameter values (including changes in the random seed) respectively.

The effect of noise on the stabilization of dynamics has been the object of extensive study, but most results were based on numerical investigations and/or were limited to low dimensional systems [Wackerbauer, 1995; Scheutzow, 1995; Reiman, 1994; Billah & Shinozuka, 1991]. A solid mathematical theorem to correlate noise to stabilization for a high-dimensional nonlinear system is lacking, although recent work seems to reveal that noise is not an unavoidable detriment but an essential element for an artificial or physical system to function well [Chang et al., 1998; Liljenström & Wu, 1995; Wackerbauer, 1995; Carroll & Pacora, 1993; Rills, ...
allows us to explore certain aspects of biological intelligence, such as to classify and retrieve input patterns according to prior learned categories.

This hybrid KIII dynamics provides a stable unpatterned basal state for the system while there is no significant stimulus. An adequate input can drive the system through a global state transition to a new patterned state. The mechanism is fast, fully parallel, under modulatory control, and flexible in absorbing new patterns from unpredictable environments. There is no requirement to follow a decision tree, or to retrieve a pattern for cross-correlation with input, or to approach an attractor asymptotically as in the descent into an energy well in using the KIII model for classification.

Empirically, the property of converging to a particular "wing" while an external input is present is independent of the hybrid KIII's initial conditions (see Fig. 6). It is possible for the application to be generalized to perform certain successive classification and retrieval tasks, that is, to study the state transitions between "wings" of a global attractor by successive stimuli (multiple \( t^{(on)} \)’s) of different classes. For example, let the time duration of each input stimulus be 200 msecs. We may start with Pattern \( X_1 \) at \( t_1^{(on)} \), then Pattern \( X_2 \) at \( t_2^{(on)} \), and so on.

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